

Due **Feb 7** by the end of the day. Just submit it on canvas. The solution has to be typed (using L^AT_EX).

1: How many ways are there to pick a man and a woman who are not husband and wife from a group of n married couples?

2: How many ternary $(0,1,2)$ sequences of length 10 are there without any two consecutive digits **being** the same?

(Ternary means using digits 0,1,2. Similarly, binary would mean just digits 0,1. No two consecutive the same means 11, 22, 00 are the forbidden substrings.)

3: We are given eight rooks, five of which are red and three of which are blue.

(a) In how many ways can the eight rooks be placed on an 8-by-8 chessboard so that no two rooks can attack one another?

(b) In how many ways can the eight rooks be placed on a 12-by-12 chessboard so that no two rooks can attack one another?

4: There are n sticks lined up in a row, and k of them are to be chosen.

(a) How many choices are there?

(b) How many choices are there if no two of the chosen sticks can be consecutive?

5: How many sets of three integers between 1 and 20 are possible if no two consecutive integers are to be in a set?

6: Combinatorially prove the following binomial identity

$$\sum_{k=0}^r \binom{n+k}{k} = \binom{n+r+1}{r}.$$

7: How many integral solutions of

$$x_1 + x_2 + x_3 + x_4 = 30$$

satisfy $x_1 \geq 2, x_2 \geq 0, x_3 \geq -5$, and $x_4 \geq 8$?

(Use substitution to get ≥ 0 for all variables.)