

MATH 484 - Exam #3 (sample) Section Name:

Date: Dec 2 2011

Start time End time:

Work on your own. Write clearly. Ask if something is not clear. If you need more paper, let me know. Good luck!

Question 1:

Write definitions of the following terms: *(Also define (P) if needed)*

- superconsistent program

- $MP(z)$, is necessarily $MP(z)$ continuous?

- Lagrangian $L(\mathbf{x}, \lambda)$

- constrained geometric program and its dual

- duality gap

- penalty parameter

- generalized penalty function

Question 2:

State theorems which give answers to the following questions: (*without proofs*)

- What can you say about $MP(z)$ if (P) is super consistent? (*Theorem 5.2.6*)

- Karush-Kuhn-Tucker Theorem (Saddle point version)

- What are sufficient conditions for a constrained geometric program (GP) to have no duality gap?
(*Theorem 5.3.5*)

- State duality theorem for Linear programming

Question 4:

Suppose you own three gas stations g_1, g_2 and g_3 . To refill them, you need to get for them 1000, 2000 and 1500 barrels of gasoline. There are two nearby gasoline storages s_1 and s_2 that you can use. The costs are \$400 for one barrel in the first one and \$415 in the second. The first storage currently has 2500 barrels of gasoline and the second has 3500 barrels of gasoline. The cost in dollars of transportation of one barrel between storages and gas stations is given in the following table:

	g_1	g_2	g_3
s_1	10	20	25
s_2	5	5	10

Formulate a linear program whose solution is minimizing the cost of refilling your gas stations. Write the dual program and explain what is the meaning of dual variables.

Question 5:

Solve the following program (P) using penalty method.

$$(P) \begin{cases} \text{Minimize } f(x) = x^2 \\ \text{subject to } g(x) = 1 - x \leq 0 \\ x \in \mathbb{R} \end{cases}$$

Question 6:

Apply the Karush-Kuhn-Tucker Theorem to locate all solutions of the following convex program:

$$(P) \begin{cases} \text{Minimize } f(x_1, x_2) = e^{-(x_1+x_2)} \\ \text{subject to } e^{x_1} + e^{x_2} \leq 20 \\ x_1 \geq 0 \end{cases}$$

Question 7:

Solve the following geometric program:

$$(GP) \begin{cases} \text{Minimize } x^{1/2} + y^{-2}z^{-1} \\ \text{subject to } x^{-1}y^2 + x^{-1}z^2 \leq 1 \\ \text{where } x > 0, y > 0, z > 0 \end{cases}$$

Question 8: D14 only (*D13 may try too if they wish*)

State and prove extended arithmetic geometric mean inequality.