Math-484 Homework #1

I will finish the homework before 11 am Aug 31 and bring it to class. If I have troubles with my work I may come to the study session on Aug 29, 5-7 pm, 145 Altgeld Hall. If I spot a mathematical mistake I will let the lecturer know as soon as possible.
I will write clearly and neatly as the grader is not an expert in cryptography. I will sign each paper of my work and indicate if I am D14 (4 hours student).

Exercise 1: *(I will check if I can use Theorems 1,2 and 3)*

Find the local and global minimizers and maximizers of the following functions:
(a) $f(x) = x^2 + 2x$
(b) $f(x) = x^2 e^{-x^2}$

Exercise 2: *(I will recall few basic definitions)*

Determine the dimension of the smallest subspace of $\mathbb{R}^4$ that contains vectors $(0,1,0,1)$, $(3,4,1,2)$, $(6,4,2,0)$ and $(-3,1,-1,3)$.

Exercise 3: *(I will recall what are determinants)*

Compute determinants of the following real matrices:

(a) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & -2 & 1 & 0 \\ 4 & a & b & 1 \\ 1 & c & d & 4 \\ 0 & 1 & -2 & 0 \end{pmatrix}$ where $a, b, c, d \in \mathbb{R}$ are parameters

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Exercise 4: *(I will recall what are eigenvalues and eigenvectors)*

Compute eigenvalues and eigenvectors of the following real matrix

$\begin{pmatrix} 2 & 6 \\ 6 & -3 \end{pmatrix}$

Exercise 5: *(I will check the definition of semidefinity and recall computing with matrices and vectors.)*

Suppose that A is a square matrix and suppose that there is another matrix B such that $A = B^T B$. Show that A is positive semidefinite.

Hint:

Recall that $\mathbf{y} \cdot B^T \mathbf{x} = B\mathbf{y} \cdot \mathbf{x}$

Exercise 6: *(I will check the definition of semidefinity more closely.* ***D14 only****)*

Suppose that A is a $n \times n$-symmetric matrix for which $a_{ii}a_{jj} - a_{ij}^2 < 0$ for some $i \neq j$. Show that A is indefinite.

Hint:

See (1.3.4)(c) in the textbook.
