Math-484 Homework #6

I will finish this homework before 11 am Oct 12 and bring it to class. If I have troubles with my work I may come to the study session on Oct 10, 5-7 pm, 145 Altgeld Hall. If I spot a mathematical mistake I will let the lecturer know as soon as possible.
I will write clearly and neatly as the grader is not an expert in cryptography. I will sign each paper of my work and indicate if I am D14 (4 hours student).

Exercise 1: *(Can I do least squares solution for not just linear regression?)*
Compute best least square fit for polynomial

$$p(t) = x_0 + x_1 t + x_2 t^2$$

and data

t_i	-2	-1	0	1	2	3	4
s_i	-5	-1	4	7	6	5	-1

Exercise 2: *(Can I compute and use least squares using QR factorization?)*
Find best least squares solution to inconsistent linear system using QR factorization.

$$\begin{pmatrix} 1 & 1 & 2 \\ 1 & 4 & 6 \\ 1 & 1 & 0 \\ 1 & 4 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 2 \\ 2 \end{pmatrix}$$

Exercise 3: *(What is minimum norm solution?)*
Find the minimum norm solution of the underdetermined linear system

$$\begin{aligned} 2x_1 + x_2 + x_3 + 5x_4 &= 8 \\ -x_1 - x_2 + 3x_3 + 2x_4 &= 0 \end{aligned}$$

Exercise 4: *(What is the projection?)*
Find vector $\mathbf{x} \in \mathbb{R}^3$ that is closest to $(1, 1, 1)$ where $\alpha, \beta \in \mathbb{R}$ and

$$\mathbf{x} = \alpha(1, 1, 2) + \beta(2, -1, 1)$$

Exercise 5: *(Do I understand definitions?)*
Let A be a matrix with linearly independent columns. Prove that:
a) $AA^\dagger A = A$
b) $A^\dagger A = (A^\dagger A)^\dagger$
c) $P_{R(A)}$ is symmetric
d) $P_{R(A)}^2 = P_{R(A)}$

Exercise 6: *(Gradient and orthogonal complements. **D14 only**)*
Let $f(\mathbf{x})$ be a function on $\mathbb{R}^n$ with continuous first partial derivatives and let M be a subspace of R^n. Suppose $\mathbf{x}^* \in M$ minimizes $f(x)$ on M. Show $\nabla f(\mathbf{x}^*) \in M^\perp$.
If, in addition, $f(x)$ is convex, then show that any $\mathbf{x}* \in M$ such that $\nabla f(\mathbf{x}^*) \in M^\perp$ is a global minimizer of $f(\mathbf{x})$ on M.
