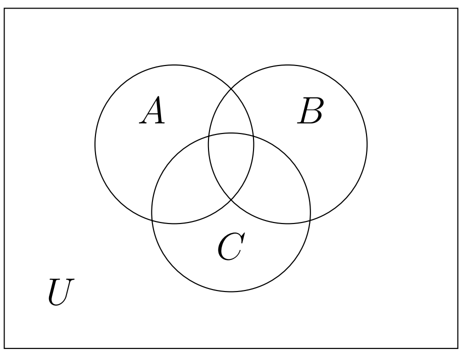
MATH213 MIDTERM 1

Feb 17 9:00-9:50am **Name: ....**

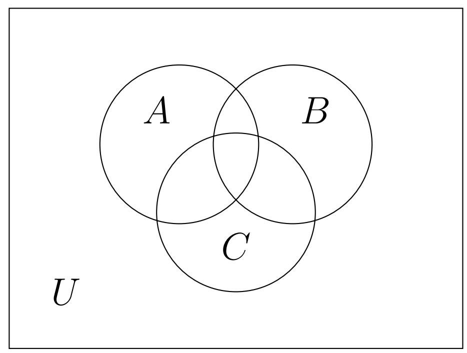
Answer as many problems as you can. Show your work. An answer with no explanation will receive no credit. Write your name on the top right corner of each page.

Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6

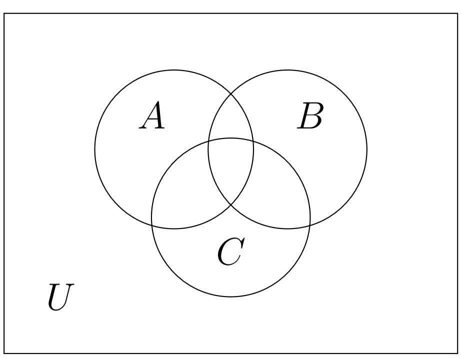
1: Draw the Venn diagrams of:



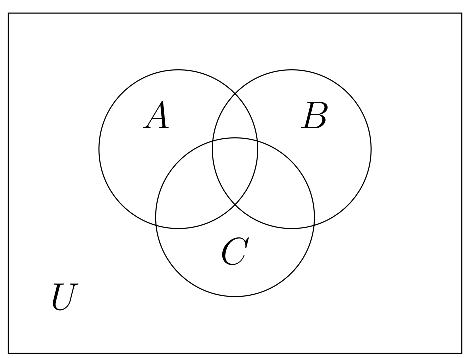
$\overline{A \cup B}$



$(A \cap B) \setminus C$



$(\overline{A \cup B \cup C}) \cup (A \cap B \cap C)$



$(A \setminus B) \cap (C \setminus B)$

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2: What is the **precise** definition of $f(x) = \omega(g(x))$?

Find the smallest integer n such that $f(x) = O(x^n)$ where
(a) $f(x) = 2x^3 + x^2 \log x$
(b) $f(x) = 3x^3 + (\log x)^4$
(c) $f(x) = 0.5x^2 + 0.4x^2 \log x$
(d) $f(x) = (x^4 + x^2 + 1)/(x^3 + 1)$
(e) $f(x) = (x^4 + 5 \log x)/(x^4 + 1)$
Justify your answer.

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3: Write the **precise** definition what does it mean that a function $f : A \to B$ is one-to-one.

Decide for the following functions if they are surjective, bijective or injective

(a) $f_1(x) = -2x^3$ where $f_1 : \mathbb{R} \to \mathbb{R}$
(b) $f_2(x) = -2x^3$ where $f_2 : \mathbb{Z} \to \mathbb{Z}$
(c) $f_3(x) = (-1)^x \lfloor x/16 \rfloor$ where $f_3 : \mathbb{N} \to \mathbb{Z}$
(d) $f_4(x) = (-1)^x \lfloor (x+1)/2 \rfloor$ where $f_4 : \mathbb{Z}^+ \to \mathbb{Z}$

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4: Draw the graphs of these functions.

(a) $f_1(x) = \lfloor x - \frac{1}{2} \rfloor + 1$;

(b) $f_2(x) = \lfloor 2x + \frac{1}{2} \rfloor - \lceil x - \frac{1}{2} \rceil$;

(c) $f_3(x) = \lfloor 0.5 \lceil 2x/3 \rceil + 0.5 \rfloor$.

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5: Show using mathematical induction that for every integer $n \geq 0$,

$$1 + 3 + 9 + 27 + \cdots + 3^n = \frac{3^{n+1} - 1}{2}.$$

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6: Which amounts of money can be formed using just two-dollar bills and five-dollar bills? Prove your answer using strong induction!
(It is not necessary to use at least one from each kind - you may use zero.)

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7: *(Bonus question)* Suppose that $\log_2 f(x) = O(\log_2 g(x))$. Show that it does not necessarily mean that $f(x) = O(g(x))$.

Describe halting problem. A precise description is required. Write also prove if you know.

Paper for attempts.