MATH213 MIDTERM 1 - sample version

Feb 17 9:00-9:50am **Name: .....**

Answer as many problems as you can. Show your work. An answer with no explanation will receive no credit. Write your name on the top right corner of each page.

Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6

1: Find $A \times B \times C$, where $A = \{a, b\}$, $B = \{1, 2, 3\}$, and $C = \{2, b\}$.

Name: .....

2: Answer whether
(a) $f(x) = x^3 + 2x$ is $O(1 + \frac{x^4}{1000})$?
(b) $f(x) = x^3 + 2x$ is $O(1000x^2 + 1)$?
Justify your answer.

Name: .....

3: Give an example of a function from $\mathbb{N}$ to $\mathbb{Z}$ that is
(a) one-to-one but not onto;
(b) onto but not one-to-one;
(c) both onto and one-to-one;
(d) neither one-to-one nor onto.

Name: .....

4: Translate each of the quantifications below into English and determine its truth value:

(a) $\exists y \in \mathbb{R}\ (2y + 2 < y)$

(b) $\exists y \in \mathbb{R}\ \forall x \in \mathbb{Z}\ (100x > -y)$

(c) $\forall x \in \mathbb{Z}\ \exists y \in \mathbb{R}\ (x > 100y)$

(d) $\exists y \in \mathbb{R}\ \forall x \in \mathbb{Z}\ (x^2 - 100 > 20y)$

Name: .....

5: Show using mathematical induction that for every positive integer n,

$$\frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 11} + \ldots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}.$$

Name: .....

6: Using mathematical induction, prove that every postage of at least 8 cents can be formed using only 5-cent and 3-cent stamps.

Paper for attempts.