MATH213 MIDTERM 2

March 16 9:00-9:50am **Name: ....**

Answer as many problems as you can. Show your work. **An answer with no explanation will receive no credit.** Write your name on the top right corner of each page.

Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6

1: How many bit strings contain exactly twenty 0s and seven 1s, if every 1 must be immediately followed by at least two 0s?

Name: ....

2: Using a combinatorial proof (**not a computational**), prove that if n and k are integers with $1 \leq k \leq n$, then

$$k(k-1)\binom{n}{k} = n(n-1)\binom{n-2}{k-2}.$$

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3: State the Multinomial theorem:

Find coefficient at

(a) $x^4y^2z^2$ in expansion of $(x+y+z)^8$

(b) x^4yz^3 in expansion of $(2x^2-3y+4z-w)^6$

(c) $x^2y^2z^2$ in expansion of $(2x+2y+2z)^5$

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4: How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 16,$$

where
a) $x_1 > 3, x_2 \geq -2, x_3 \geq 1$ and $x_4 > 5$ are integers.
b) $6 > x_1 \geq 1, x_2 \geq -1, x_3 \geq 5$ and $x_4 \geq -3$ are integers.

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5: State the Pigeonhole principle:

A child watches TV at least one hour each day for seven weeks but, because of parental rules, never more than 11 hours in any one week. Prove that there is some period of consecutive days in which the child watches exactly 20 hours of TV. (It is assumed that the cild watches TV for a whole number of hours each day.)

Name: ....

6: How many four-digit integers n satisfy **all** of the following conditions:?

(i) $n > 40000$.

(ii) the digits are distinct.

(iii) n is even.

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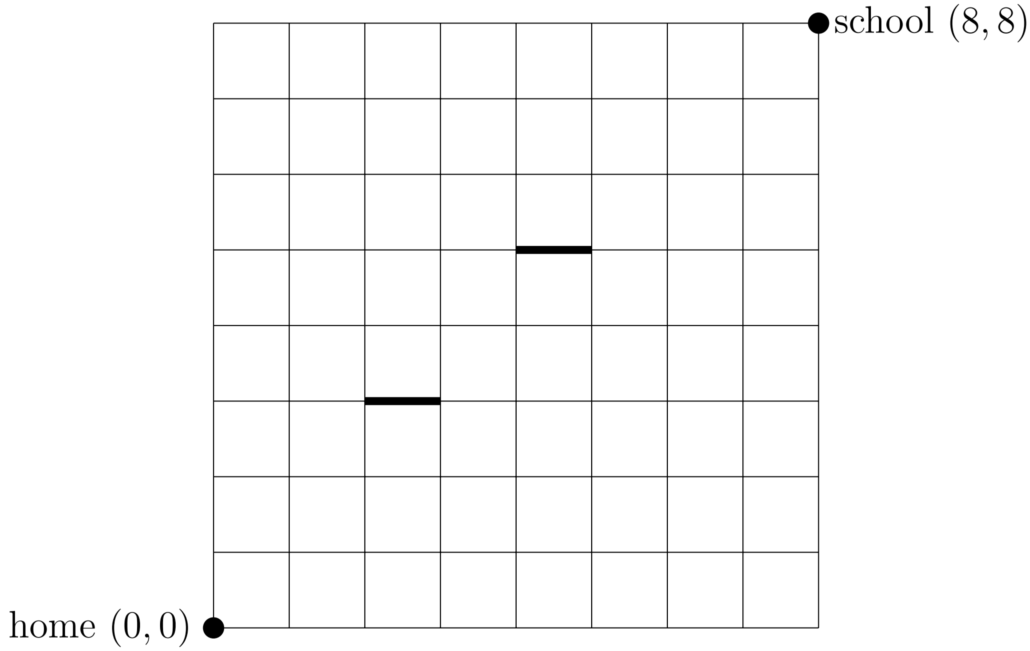
7: A student has a lecture in a building located nine blocks east and eight block north of his home. Every day he walks 16 blocks to school. How many different routes are possible for him if

(a) there are no additional constraints.

(b) routes must pass through **both** of the thick edges.

(c) routes must **not** pass through any of the two thick edges.



(In the picture walking is along edges.*)*

Paper for attempts.