MATH213 MIDTERM 2 - Sample version

March 16 9:00-9:50am **Name: ....**

Answer as many problems as you can. Show your work. **An answer with no explanation will receive no credit.** Write your name on the top right corner of each page.

Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6

1: How many positive integers between 100 and 999 (including both numbers)

(a) are divisible by 3 and 4?

(b) are divisible by 3 or 4?

(c) are divisible by 3 but not by 4?

(d) are divisible by 4 but not by 3?

(e) are divisible neither by 4 nor by 3?

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2: Find the least number of cables required to connect 100 computers to 20 printers to guarantee that every subset of 20 computers can directly access 20 different printers. Justify your answer.

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3: How many permutations of the letters ABCDEFG contain

a) the string BCD?

b) the string CFGA?

c) the strings BA and GF?

d) the strings ABC and DE?

e) the strings ABC and CDE?

f) the strings CBA and BED?

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4: State the Binomial theorem:

Find coefficient at

a) x^4y^2 in expansion of $(x+y)^6$
b) x^4y^2 in expansion of $(2x-3y)^6$
c) x^5y^2 in expansion of $(-x+y^2)^6$
d) x^4y^2 in expansion of $(-x^3+y)^6$

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5: How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 13,$$

where
a) $x_1 > 1, x_2 \geq -1, x_3 \geq 0$ and $x_4 > 3$ are integers.
b) $3 > x_1 \geq 1, x_2 \geq -2, x_3 > -1$ and $x_4 \geq 2$ are integers.

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6: My sister spends at least one hour a day on Facebook, but never more than 10 hours a week. She is always using it a whole number of hours a day. Prove that if she does this for nine weeks, there is some number of consecutive days where she spent exactly 35 hours on Facebook.

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7: How many four-digit integers n satisfy **all** of the following conditions:?

(i) $n > 5000$.

(ii) the digits are distinct.

(iii) n is odd.

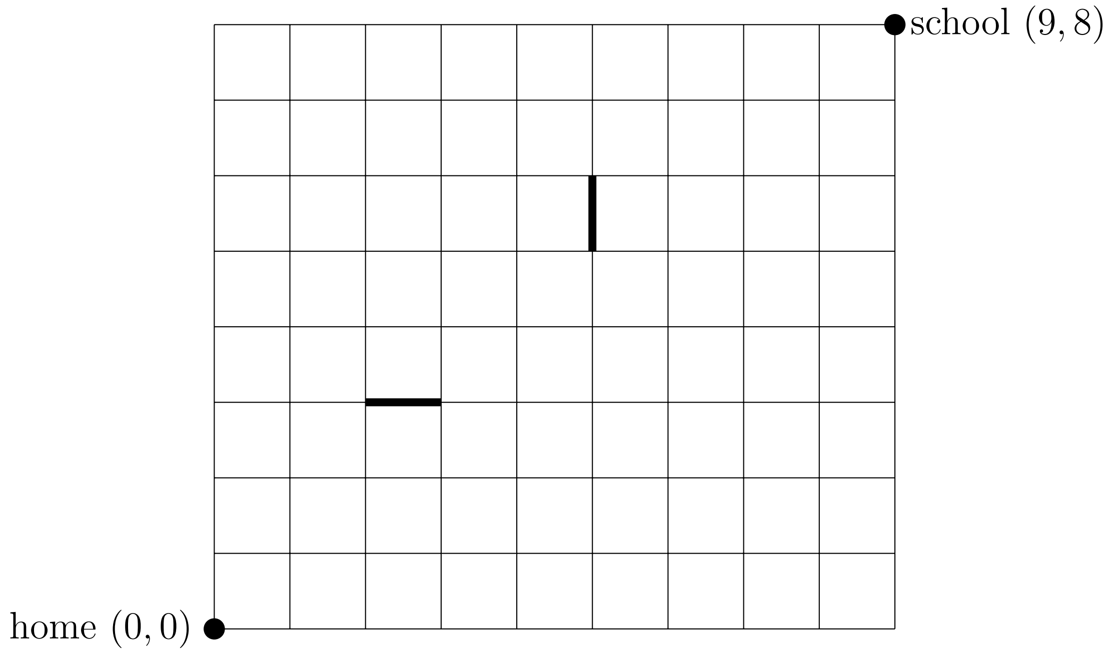
Name: ....

8: A student has a lecture in a building located nine blocks east and eight block north of his home. Every day he walks 17 blocks to school. How many different routes are possible for him if

(a) there are no additional constraints.

(b) routes must pass through **both** of the thick edges.

(c) routes must **not** pass through any of the two thick edges.



(In the picture walking is along edges.*)*

Name: ....

9: Combinatorially prove the following binomial identity

$$\sum_{k=0}^{r} \binom{n+k}{k} = \binom{n+r+1}{r}.$$

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10: There are 20 identical knights lined up in a row occupying 20 distinct places as follows:

♘♘♘♘♘♘♘♘♘♘♘♘♘♘♘♘♘♘♘♘

Six of them will be replaced by rooks ♖. How many possible replacements are there if no two of the rooks can be next to each other?

Paper for attempts.