MATH213   MIDTERM 3

April 27 9:00-9:50am   **Name:** ....
Answer as many problems as you can. Show your work. Counted will be 5 out of 6 questions. **An answer with no explanation will receive no credit.**

Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6

1:   Result of an anonymous survey was that 80% of students know how to swim. University decided that all students should be able to swim. So they created a swimming test. To pass the swimming test, student must be able to swim 100 meters in one minute. 90% of students who can swim pass the test. But students who cannot swim fail the test with probability 99%. If a student failed the test, what is the probability that he(she) can swim? Compute also what is the probability that he(she) cannot swim if he(she) failed the test.

2: A class of 30 students went for a hike. The trail was a bit dirty. So 13 students returned with dirty trousers, 13 have dirty T-shirts and 10 have dirty shoes. Both trousers and T-shirt were dirty on 5 students, T-shirt and shoes were not clean on 4 students and both trousers and boots were dirty on three students. Moreover, there are 5 students who remained entirely clean. Was there anyone entirely dirty?

3: Solve the following homogeneous recurrence relations:

$$a_n = -a_{n-1} + 6a_{n-2} \text{ for } n \geq 2, a_0 = 1, a_1 = 0$$

and

$$b_n = 11b_{n-1} - 30b_{n-2} \text{ for } n \geq 2, b_0 = 0, b_1 = 1$$

4: Draw simple graphs satisfying the following conditions or argue that no such graph exist:
(a) degree sequence 1,1,1,2,2,3 and connected
(b) degree sequence 1,1,1,2,2,3 and two connected components
(c) degree sequence 1,2,3,4,5,6
(d) degree sequence 0,1,2,3,4,5

5:   (a) For what values of m and n does $K_{n,m}$ have an Euler cycle?
(b) For what values of m and n does $K_{n,m}$ have a Hamilton path?

6:   Let G be a connected simple planar graph with e edges, v vertices and f faces. Assume that G has a drawing that does not contain any faces of size three or four. Prove that
$$e \leq \frac{5v - 10}{3}.$$