MATH213 MIDTERM 3 - Sample version

April 27 9:00-9:50am **Name: .....**

Answer as many problems as you can. Show your work. Counted will be 5 out of 6 questions. **An answer with no explanation will receive no credit.**

Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6

1: *(Probability - general)* What is the probability that six consecutive integers will be chosen as the winning numbers in a lottery where each of six numbers chosen is an integer between 1 and 40 (inclusive)? *(Lottery randomly chooses 6 different numbers from 1 to 40 where order of the numbers does not matter)*

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2: *(Probability - general)* Let $\mathcal{E}$ be a collection of events in a probability space.
(a) Define what does it mean that elements of $\mathcal{E}$ are mutually independent.
(b) Is it true that if every pair of events from $\mathcal{E}$ is independent, then events in $\mathcal{E}$ are mutually independent? (Give proof or a counterexample)

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3: *(Probability - Bayes theorem)* Suppose that a Bayesian spam filter is trained on a set of 500 spam messages and 200 messages that are not spam. The word *diploma* appears in 40 spam messages and in 25 messages that are not spam. Would an incoming message be rejected as spam if it contains the word *diploma* and the threshold for rejecting spam is 0.9?

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4: *(Principle of inclusion and exclusion)* How many solutions does the equation

$$x_1 + x_2 + x_3 = 13$$

have where x_1, x_2, and x_3 are nonnegative integers less than 6?

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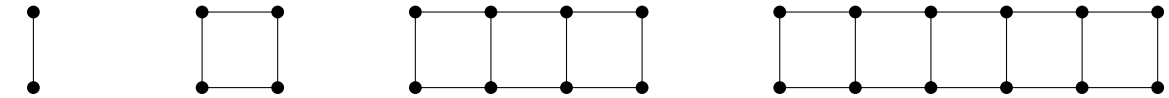
5: *(Recurrence relations)* Count the number of perfect matchings of points of an earthworm W_{2n} on $2n$ vertices when only segments like in the picture may be used in the matching.



W_2 W_4 W_8 W_{12}

For example W_4 has two perfect matchings (a) and (b):

(a) (b)

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6: *(Graph theory)* Draw a simple graph with the following sequence of degrees such that it has exactly two connected components or argue that no such graph exists:
(a) 1,1,1,1,1,1
(b) 1,1,1,1,2,2
(c) 1,1,2,2,2,2
(d) 2,2,2,2,2,2
(e) 2,2,2,2,3,3

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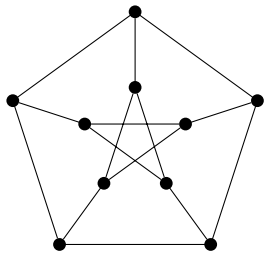
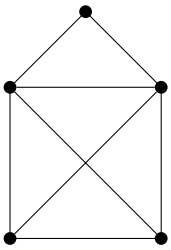
7: Does the following graphs have:
(a) Hamilton path
(b) Hamilton circuit
(c) Eulerian trail (path)
(d) Eulerian cycle
(e) perfect matching
Either find one or argue why no such graph exists. Determine also their
(f) edge connectivity
(g) vertex connectivity



(A)



(B)

Name: .....

8: Let G be a connected simple planar graph with e edges, v vertices and f faces. Assume that G does not contain any triangles. Prove that

$$e \leq 2v - 4.$$

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9: Describe Dijkstra's algorithm. Find shortest path in the following graph using Dikstra's algorithm from vertex u to vertex v in the graph below. (Show steps of the algorithm, not just the result!)