

MATH213 HW 1

due **Feb 1** before class

1: For each of the following sets, determine whether 3 is an element of that set and also whether $\{3\}$ is an element of that set.

- (a) $\{4, 3, \{4\}, 3\}$,
- (b) $\{4, \{3\}, \{4\}, \{\{3\}\}\}$,
- (c) $\{4, \{3, 4\}, \{4\}, \{3, \{3\}\}\}$,
- (d) $\{4, \{3, 3\}, \{4\}, 3, \{\{3\}\}\}$,
- (e) $\{4, \{\{3\}\}, \{4\}, \{\{3\}\}\}$.

2: Determine whether each of the statements below is true or false.

- (a) $0 \in \{\emptyset\}$;
- (b) $\{\emptyset\} = \emptyset$;
- (c) $\emptyset \subset \{0\}$;
- (d) $\{\emptyset, \emptyset\} \subseteq \{\emptyset\}$;
- (e) $\{\{\emptyset\}\} \subseteq \{\emptyset, \{\emptyset\}\}$;
- (f) $\{\{\emptyset\}\} \in \{\emptyset, \{\emptyset\}\}$.

3: Let $A = \{x\}$, $B = \{1, 2, 3\}$, and $C = \{b, c\}$. Find

- (a) $B \times C$;
- (b) $A \times C \times B$;
- (c) $C \times B \times C$.

4: Translate each of the quantifications below into English and determine its truth value:

- (a) $\exists y \in \mathbf{R} (y + 2 > y)$
- (b) $\forall x \in \mathbf{Z} \exists y \in \mathbf{R} (x - 1 > y)$
- (c) $\exists y \in \mathbf{R} \forall x \in \mathbf{Z} (x - 1 > y)$
- (d) $\exists y \in \mathbf{R} \forall x \in \mathbf{Z} (x^2 > y)$

5: Let A, B , and C be sets. Show (in two ways: with the help of membership tables and by arguing like in Examples 10 and 12 from Chapter 2) that

- (a) $\overline{A - B - C} = \overline{A} \cup B \cup C$
- (b) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

6: Let $A = \{2, 4, 6, 8, 10, 11\}$, $B = \{1, 2, 4, 8, 9\}$, and $C = \{1, 3, 6, 8, 9, 11\}$.

Find

(a) $A \cap B \cap C$;

(b) $(A \cup B) - C$;

(c) $(A \cap B) \cup C$.

In each case write the corresponding bit string (characteristic vector) of length 11.