

## MATH213      HW 3

due **Feb 15** before class

**1:** Use the bubble sort and the insertion sort algorithms to sort 5, 3, 9, 7, 1 showing the lists obtained at each step. (*Insertion sort is on page 198, or see Wikipedia.*)

**2:** Determine whether each of these functions is  $O(x^3)$  and whether it is  $o(x^3)$ :

(a)  $f_1(x) = 1000 + 10x^2 + \frac{x^3}{1+x}$ ;

(b)  $f_2(x) = -1000 + x^2 + x\frac{x^3}{1+x}$ ;

(c)  $f_3(x) = \frac{2^x}{x^9} - x^3$ ;

(d)  $f_4(x) = 3^{\log_2 x}$ .

( $f(x) = o(g(x))$  means that  $f(x)$  grows much less than  $g(x)$ . Precise definition is that for every  $c$  exists  $x_0$  such that for all  $x \geq x_0$  holds  $f(x) < cg(x)$ . Notice that  $f(x) = o(g(x))$  iff  $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 0$ . See Wikipedia or page 218 from the book for more details.)

**3:** For functions  $f(x) = \frac{x^2}{3-x} \log_3 x + \sqrt{x}$  and  $g(x) = \frac{x^5+x}{1+x+x^4}$ , determine whether

$f(x) = O(g(x))$ ,  $f(x) = \Omega(g(x))$ , or  $f(x) = \Theta(g(x))$ .

**4:** Use mathematical induction to prove that for harmonic numbers  $H_k = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{k}$ , the inequality  $H_{2^n} \leq 1 + n$  holds for every nonnegative integer  $n$ .

**5:** Use mathematical induction to prove that  $4n^3 + 5n$  is divisible by 3 for every nonnegative integer  $n$ .

**6:** Use mathematical induction to prove that  $n^2 - 1$  is divisible by 8 for every positive odd integer  $n$ .