MATH213   HW 6

due **Mar 14** before class.
Solutions without explanation will receive no points.

1: A professor writes 40 discrete mathematics true/false questions. Exactly 17 of these statements are true. If the questions can be positioned in any order, how many different answer keys are possible.

2: There are n sticks lined up in a row, and k of them are to be chosen.
(a) How many choices are there?
(b) How many choices are there if no two of the chosen sticks can be consecutive?

3: A secretary works in a building located nine blocks east and eight block north of his home. Every day he walks 17 blocks to work. (See the map that follows.)
   (a) How many different routes are possible for him?
   (b) How many different routes are possible if the one block in the easterly direction, which begins four block wast and three blocks north of his home, is under water (and he can't swim)?
(Hint: use subtraction principle)
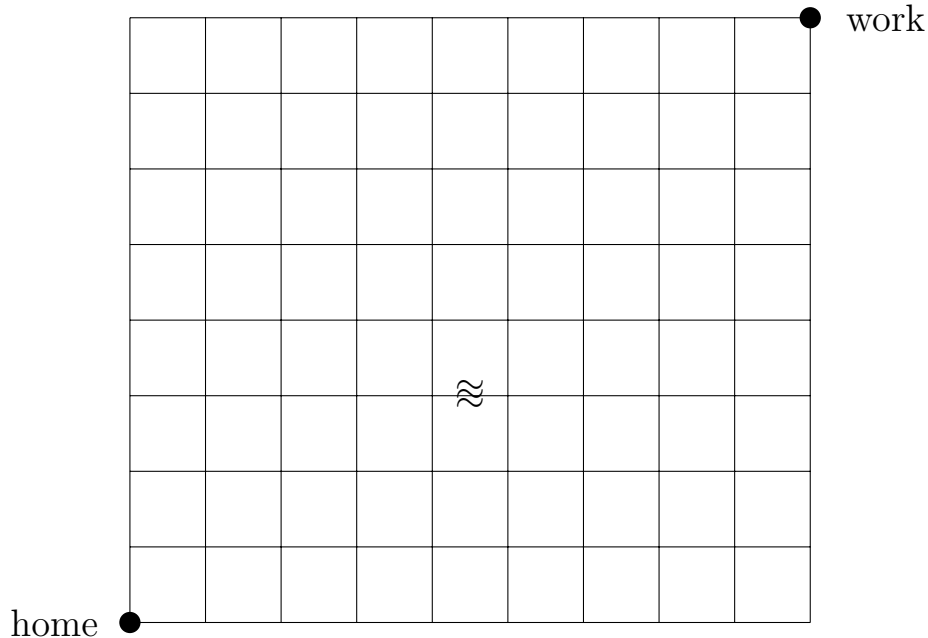
4: A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose

(a) a dozen croissants?

(b) two dozen croissants with at least two cherry croissants and at least three croissants of each other kind?

(c) a dozen croissants with at most 3 plain croissants and at least two cherry croissants?

(d) two dozen croissants with exactly 2 cherry croissants, at most 4 plain croissants and at least 3 apple croissants?

5: A video game store chain has 1000 copies of Legend of Zelda bundled with golden remote controller. How many ways are there to store the video games in their three warehouses if the copies are indistinguishable?

6: How many solutions are there to the inequality

$$x_1 + x_2 + x_3 \leq 13,$$

where x_1, x_2, and x_3 are nonnegative integers such that

(a) $x_1 \geq 2$?

(b) $x_1 \leq 2$ and $x_3 \geq 3$?
(Hint: introduce an auxiliary variable x_4 so that $x_1 + x_2 + x_3 + x_4 = 13$.)

7: How many different strings can be made from the letters in MATHE-MATICS, using all letters?

8: How many ways are there to travel in 3-dimensional space with axis x y and z from the origin $(0, 0, 0)$ to the point $(4, 3, 5)$ by moving in steps where each step is move in direction of one of the axis by 1? Moving in negative direction is prohibited. You may also view it as moving on 3-dimensional grid with unit segment sizes. *(Hint: total number of steps in any path is 12.)*

9: Find the coefficient at $x^2 y^3 z^2$ in the expansion of $(2x + 3y - 4z)^7$.

10: How many ways can 30 books be placed on four distinguishable shelves
(a) if all books are indistinguishable copies of the same title?
(b) if all books are distinguishable, and the order of the books on a shelf does NOT matter?
(c) if 10 books are indistinguishable copies of the same title and all other books are distinguishable and the order of the books on a shelf does NOT matter?

11: What is the probability that a five-card poker hand contains
(a) a straight, that is, five cards that have consecutive kinds (for example, Q-J-10-9-8)?
(b) two pairs, that is, two of each of two different kinds and a fifth card of a third kind?