MATH213 HW 7

due **Apr 4** before class.
Solutions without explanation will receive no points.

1: What is the probability of these events when we randomly choose a permutation of $\{1, 2, \ldots, n\}$ where $n \geq 4$?
(a) 2 precedes n.
(b) 1 precedes 2 and 2 precedes n.
(c) 1 immediately precedes 3.
(d) 1 immediately precedes 3, and n immediately precedes 2.

2: Suppose that E_1 and E_2 are events such that $p(E_1) = 3/4$ and $p(E_2) = 4/5$. Show that $11/20 \leq p(E_1 \cap E_2) \leq 3/4$.

3: A student takes a random card from a deck of 52 cards, looks at it and puts it back. If he does this k times, what is the probability that he took a card of some kind at least twice? What is the smallest k such that this probability is greater than $1/2$?

4: Suppose that E and F are events in a sample space and that $p(E) = 1/3$, $p(F) = 1/2$, and $p(E|F) = 2/5$. Find $p(F|E)$.

5: When a test for steroids is given to soccer player, 98% of players taking steroids test with positive result and 12% of the players not taking steroids have positive test result. Suppose that 5% of soccer players really take steroids. What is the probability that the soccer player who has a positive test really takes steroids?

6: A space probe near Neptune communicates with Earth using bit strings. Suppose that in its transmissions it sends a 1 one-third to the time and a 0 two-thirds of the time. When a 0 is sent, the probability that it is received correctly is 0.9, and the probability that it is received incorrectly (as a 1) is 0.1. When a 1 is sent, the probability that it is received correctly is 0.8, and the probability that it is received incorrectly (as a 0) is 0.2.
a) Find the probability that a 0 is received.

b) Use Baye's theorem to find the probability that a 0 was transmitted given that a 0 was received.

7: Let h_n equal the number of different ways in which the squares of a $1 \times n$ chessboard can be colored using the colors white, red, and blue in such a way that no two squares that are colored red are adjacent. Find and verify a recurrence relation for h_n. Then find a formula for h_n.

8: Solve the recurrence relation $h_n = 4h_{n-2}$, $(n \geq 2)$ with initial values $h_0 = 0$ and $h_1 = 1$.

9: Solve the recurrence relation $h_n = 8h_{n-1} - 16h_{n-2}$, $(n \geq 2)$ with initial values $h_0 = -1$ and $h_1 = 0$.