

MATH213 HW 9

due Apr 18 before class.

Solutions without explanation will receive no points.

1: How many vertices and how many edges do these graphs have?

a)  $K_n$  b)  $C_n$  c)  $W_n$  d)  $K_{m,n}$  e)  $Q_n$

Draw these graphs for  $n = 4$  and  $m = 2$ .

2: Determine whether each of these sequences is graphic (is the list of degree of vertices of some graph). For those that are, draw a graph having the given degree sequence.

a) 5, 4, 3, 2, 1, 0 b) 6, 5, 4, 3, 2, 1 c) 2, 2, 2, 2, 2, 2

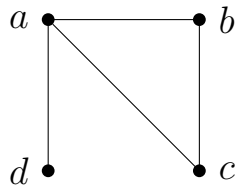
d) 3, 3, 3, 2, 2, 2 e) 3, 3, 2, 2, 2, 2 f) 1, 1, 1, 1, 1, 1

g) 5, 3, 3, 3, 3, 3 h) 5, 5, 4, 3, 2, 1

3: Show that in a simple graph with at least two vertices must be two vertices that have the same degree.

4: If the simple graph  $G$  has  $v$  vertices and  $e$  edges, how many edges does its complement  $\overline{G}$  have?

5: Draw all subgraphs and induced subgraphs of the following graph:



6: Suppose that there is an integer  $k$  such that every man on a desert island is willing to marry exactly  $k$  of the women on the island and every woman is willing to marry exactly  $k$  of the men. Also, suppose that a man is willing to marry a woman if and only if she is willing to marry him. Show that it is possible to match the men and women on the island so that everyone is matched with someone that they are willing to marry. (Hint: Use Hall's theorem)