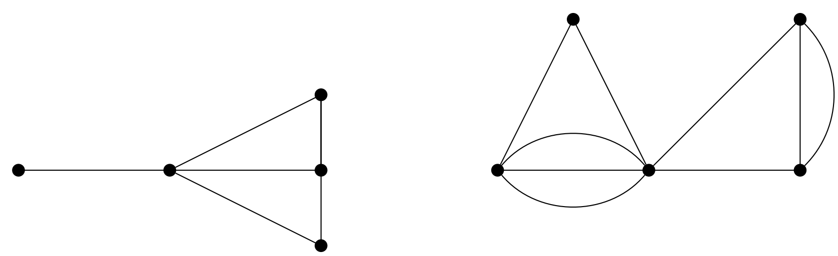
MATH213 HW 10

due **Apr 25** before class.

Solutions without explanation will receive no points.

1: Let G be a graph with exactly two vertices x and y of odd degree that are not adjacent. (All other vertices are of even degree.) Let graph G^* be obtained from G by adding edge new $\{x, y\}$ joining vertices x and y. Prove that G is connected if and only if G^* is connected.
(Hint: Degree sums.)

2: Find adjacency matrices and incidence matrices of the graphs below

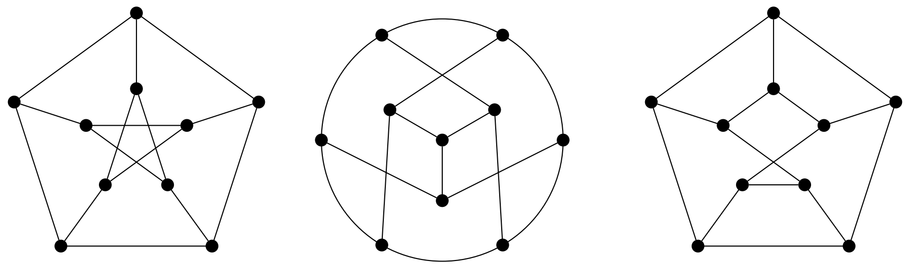


(a) (b)

3: (a) Draw the graph if $A = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$ is its adjacency matrix.

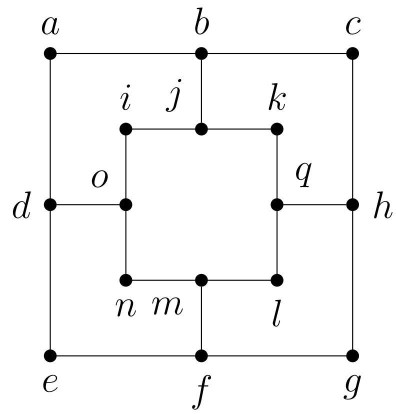
(b) Draw the graph if the same matrix A is its incidence matrix.

4: Determine which pairs of graphs below are isomorphic. Prove your statements! Just claims do not count.

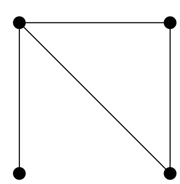


(a) (b) (c)

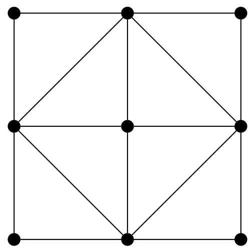
5: Determine whether the following graph has a Hamilton circuit. If it does, find duch a circuit. If it does not, dive an argument to show why no such circuit exists.



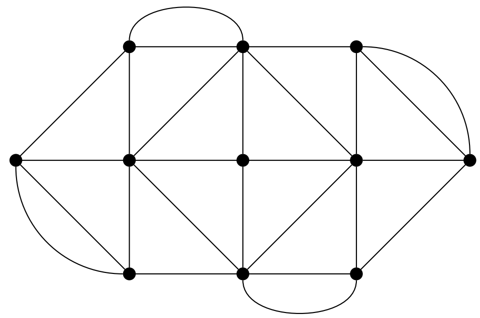
6: Draw all nonisomorphic subgraphs on 4 vertices of the following graph:



7: Determine if the following multigraphs have
(a) Hamilton circuit (b) Hamilton path (c) Eulerian path (d) Eulerian circuit
(e) Determine the minimum number of edges that has to be added to the graphs such they have Eulerian circuits



(i)



(ii)

(c,d) The graph has neither of these because it has four vertices of odd degree. In order to make all degrees even, at least two edges must be added to the graphs and then it will have an Eulerian path.

(ii) Has Hamilton cycle as depicted below. Hence it also has a Hamiltonina path. Because it has all vertices of even degree, it has Eulerian cycle as well as Eulerian path.