MATH413 HW 4

due **Feb 29** before class

1: *(P. 155, #8)* Use binomial theorem to prove that

$$2^n = \sum_{k=0}^{n} (-1)^k \binom{n}{k} 3^{n-k}$$

2: *(P. 155, #11)* Use **combinatorial** reasoning to prove the identity (in the given form)

$$\binom{n}{k} - \binom{n-3}{k} = \binom{n-1}{k-1} + \binom{n-2}{k-1} + \binom{n-3}{k-1}$$

3: *(P.155 , #12)* Let n be a positive integer. Prove that

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k}^2 = \begin{cases} 0 & \text{if } n \text{ is odd} \\ (-1)^m \binom{2m}{m} & \text{if } n = 2m. \end{cases}$$

Hint: consider $(1-x^2)^n = (1+x)^n(1-x)^n$

4: *(P. 156, #18)* Evaluate the sum

$$1 - \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} - \frac{1}{4}\binom{n}{3} + \cdots + (-1)^n \frac{1}{n+1}\binom{n}{n}.$$

5: *(P. 156, #28)* Let n and k be a positive integers. Give a combinatorial proof that

$$\sum_{k=1}^{n} k \binom{n}{k}^2 = n \binom{2n-1}{n-1}.$$

6: *(P. 158, #30)* Prove that the only antichain of $S = \{1, 2, 3, 4\}$ of size 6 is the antichain of all 2-subsets of S.