

MATH413 HW 6

due **Mar 14** before class

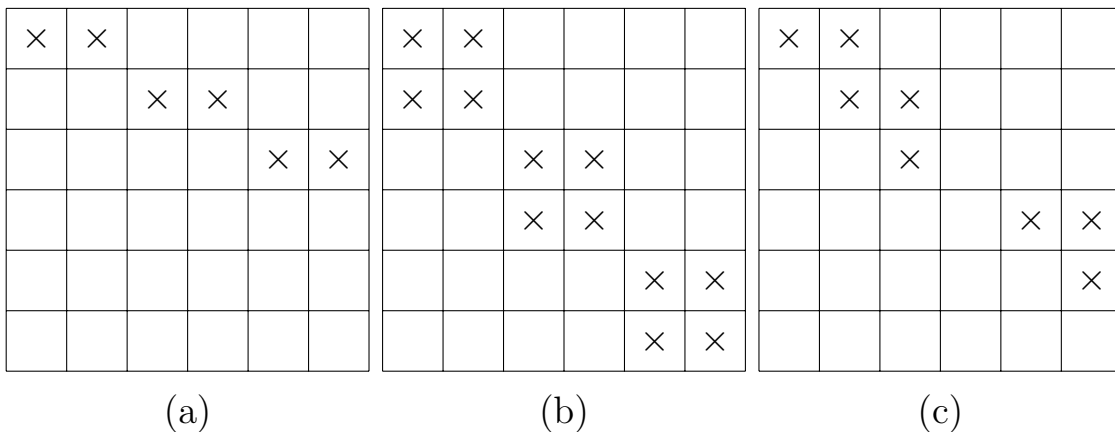
1: (*P. 199, #13*) Determine the number of permutations of $\{1,2,\dots,9\}$ in which at least one odd integer is in its natural position.

2: (*P. 199, #17*) Determine the number of permutations of the multiset

$$S = \{3 \cdot a, 4 \cdot b, 2 \cdot c\},$$

where, for each type of letter, the letters of the same type do not appear consecutively. (Thus *abbbbcaca* is not allowed but *abbacacb* is.)

3: (*P. 200, # 24*) What is the number of ways to place six nonattacking rooks on the 6-by-6 boards without forbidden positions as shown?



4: (*P. 201, # 27.*) A carousel has eight seats, each representing a different animal. Eight girls are seated on the carousel facing forward (each girl looks at another girl's back). In how many ways can the girls change seats if that each has a different girl in front of her. How does the problem change if all the seats are identical?

5: (*P. 200, #32*) Let n be a positive integer and let p_1, p_2, \dots, p_k be all the different prime numbers that divide n . Consider the Euler function ϕ defined by

$$\phi(n) = |\{k : 1 \leq k \leq n, \text{GCD}\{k, n\} = 1\}|.$$

Use the inclusion-exclusion principle to show that

$$\phi(n) = n \prod_{i=1}^k \left(1 - \frac{1}{p_i}\right).$$

6: (*P. 257, #1*) Let $f_0, f_1, f_2, \dots, f_n, \dots$ denote the Fibonacci sequence. By evaluating each of the following expressions for small values of n , conjecture a general formula and then prove it, using mathematical induction and the Fibonacci recurrence:

- (a) $f_1 + f_3 + f_5 + \dots + f_{2n-1}$
- (b) $f_0 + f_2 + f_4 + \dots + f_{2n}$
- (c) $f_0 - f_1 + f_2 - f_3 + \dots + (-1)^n f_n$
- (d) $f_0^2 + f_1^2 + f_2^2 + \dots + f_n^2$

7: (*P. 257, #3*) Prove the following about the Fibonacci numbers:

- (a) f_n is even if and only if n is divisible by 3.
- (b) f_n is divisible by 3 if and only if n is divisible by 4.
- (c) f_n is divisible by 4 if and only if n is divisible by 6.