MATH413   MIDTERM 1

Feb 17 10:00-10:50am   **Name:** ....

Answer as many problems as you can. Show your work. An answer with no explanation will receive no credit. Write your name on the top right corner of each page.

Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6

1:   How many ways can be placed 7 pairwise non-attacking rooks (♖) on a 15×15-chessboard if
(a) all rooks are the same?
(b) 2 rooks are blue, 3 are red and 2 are white?
(two different questions)

2: A student has a lecture in a building located nine blocks east and eight block north of his home. Every day he walks 17 blocks to school. How many different routes are possible for him if
  (a) there are no additional constraints.
  (b) routes must pass through **both** of the thick edges.
  (c) routes must **not** pass through any of the two thick edges.

school $(9, 8)$

home $(0, 0)$
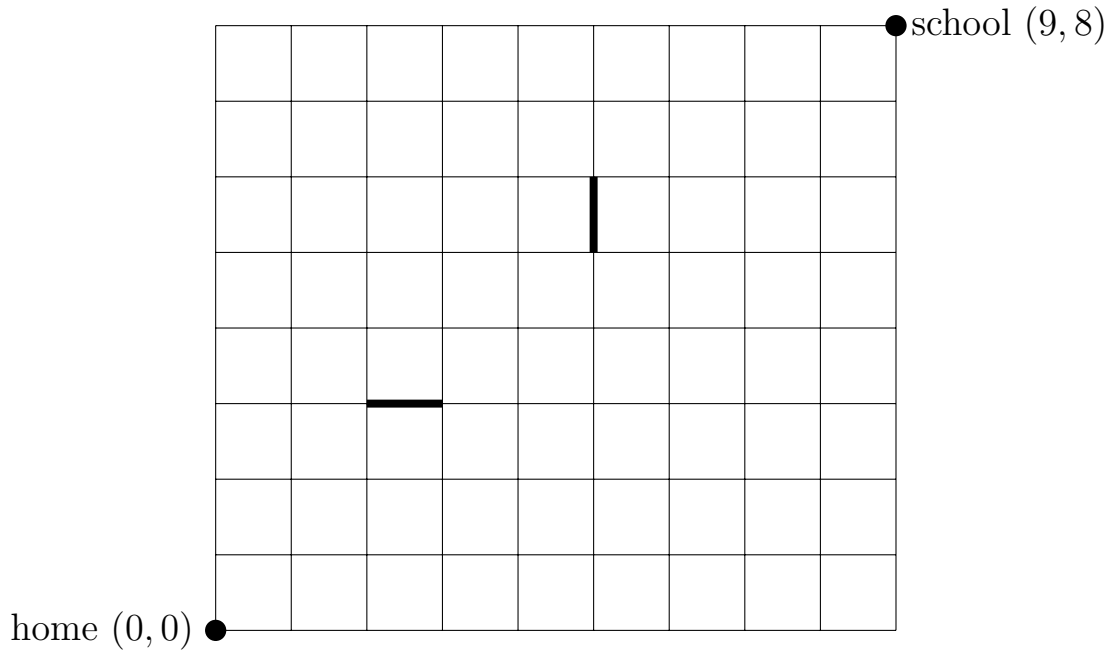
(In the picture walking is along edges.)

3: My sister spends at least one hour a day on Facebook, but never more than 10 hours a week. She is always using it a whole number of hours a day. Prove that if she does this for nine weeks, there is some number of consecutive days where she spent exactly 35 hours on Facebook.

4: How many integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 25,$$

where $x_1 \geq 2$, $x_2 \geq -1$, $0 \leq x_3 \leq 2$, $x_4 \geq 4$, and $x_5 \geq 0$?

5:   How many four-digit integers n satisfy **all** of the following conditions:?
(i) $n > 5000$.
(ii) the digits are distinct.
(iii) n is odd.

6:   There are $2n + 1$ identical books to be put in a bookcase with three distinguishable shelves. In how many ways can this be done if each pair of shelves together contains more books then the other?

(Only the number of books in each shelve gives a different way of putting. Substraction principle might help)

7: *(Bonus question)* A collection of subsets of $\{1, 2, \ldots, n\}$ has the property that each pair of subsets has at least one element in common. Prove that there are at most 2^{n-1} subsets in the collection.

Paper for attempts.