MATH413 MIDTERM 1 - sample version

Feb 17 10:00-10:50am **Name: ....**

Answer as many problems as you can. Show your work. An answer with no explanation will receive no credit. Write your name on the top right corner of each page.

Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6

1: Prove that in a group of $n > 1$ people there are two who have the same number of acquaintances in the group. (It is assumed that no one is acquainted with oneself.)

Name: ....

2: Combinatorially prove the following binomial identity

$$\sum_{k=0}^{r} \binom{n+k}{k} = \binom{n+r+1}{r}.$$

Name: ....

3: There are 20 identical knights lined up in a row occupying 20 distinct places as follows:

♘♘♘♘♘♘♘♘♘♘♘♘♘♘♘♘♘♘♘♘

Six of them will be replaced by rooks ♖. How many possible replacements are there if no two of the rooks can be next to each other?

Name: ....

4: Determine the number of ways to distribute 10 orange drinks, 1 lemon drink, and 1 strawberry drink to four thirsty students so that each student gets at least one drink, and the lemon and lime drinks go to different students.

Name: ....

5: How many five-digit integers n satisfy **all** of the following conditions:? (first digit cannot be 0)

(a) $n > 60000$.

(b) the digits are distinct.

(c) n is even.

Name: ....

6: Assume that in a lottery 7 numbers are chosen out of $1 \ldots 90$. (order of choice does not matter)
(i) How many ways can it be done, that all the numbers are even?
(ii) How many ways can it be done, that there are exactly 3 even numbers chosen?
(iii) How many ways can it be done, that there are more even numbers than odd numbers?
(iv) How many ways can it be done, that there are no two consecutive numbers chosen?

Paper for attempts.