

MATH413 MIDTERM 3

April 27 10:00-10:50am

Name:

Answer as many problems as you can. Best 5 out of 6 counts. Show your work. An answer with no explanation will receive no credit. Write your name on the top right corner of each page.

Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6

1: Without using generator functions, determine h_n , where

for every $n > 1$ $h_n = 5h_{n-1} - 6h_{n-2} - 4n + 14$, $h_0 = 7$, $h_1 = 14$.

Name:

2: Determine the generating function for the sequence $\{h_n\}_{n=0}^{\infty}$ that satisfies the relation $h_n = 6h_{n-1} - 8h_{n-2}$ for $n \geq 2$ with initial conditions $h_0 = 1, h_1 = 0$. Using the generating function find an explicit formula for h_n in this problem.

Name:

3: Determine the number h_n of n -digit numbers with each digit odd, where the digits 1 and 3 occur an even number of times.

Name:

4: Using the difference sequence method, give a close formula for

$$\sum_{k=1}^n 2k^3 - 3k^2 + k + 2.$$

(notice that the sum is from $k = 1$)

Name:

5: Prove that for any integer n the number of partitions where no part appears more than three times equals the number of partitions where no part is a multiple of four.

Name:

6: A Stirling's birthday party is tonight! All $2n$ invited guests are excited to come and contribute \$5 to a present for Stirling - a very expensive bottle of wine from Catalonia (in Spain). Fibonacci volunteered to collect the contributions from all the guests. Somehow happened that n guests have exact \$5 bills but the other n guests have only \$10 bills (and want to get \$5 back). Help Fibonacci to compute in how many ways he can order collecting money from $2n$ (distinguishable!) guests such that he never runs into a trouble with giving change back.

(If Fibonacci had n -times \$5 bill then there would be $(2n)!$ possible orderings of the guests. But he has no \$5 bill.)