Math-484 Homework #5 (Least squares)

I will finish the homework before 10am Oct 10. If I spot a mathematical mistake I will let the lecturer know as soon as possible.
I will write clearly and neatly as the grader is not an expert in cryptography. I will sign each paper of my work and indicate if I am C14 (4 hours student).

1: (*Do I know how to solve inconsistent system?*)
Find the least squares solution of the inconsistent linear system of 6 equations:

$$x_1 + x_2 + x_3 = 3 \qquad\qquad x_3 = 1$$
$$x_1 + x_3 = 2 \qquad\qquad 2x_1 + 5x_3 = 8$$
$$-7x_1 + 8x_2 = 0 \qquad\qquad x_1 + 2x_2 - x_3 = 1$$

2: (*Can I do understand linear regression?*)
Compute the equation of the linear regression line corresponding to the data on the table below:

x	-2	-1	0	1	2	3
y	12	11	8	5	2	-3

3: (*How much can the solution change?*)
Compute and compare solutions of the two systems

$$\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0.01 \end{pmatrix}.$$

4: (*How do I compute generalized inverse?*)
Compute generalized inverz $A^\dagger$ of

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 2 \\ 2 & 3 \end{pmatrix}$$

5: (*Am I familiar with Gram-Schmidt Process?*)
Find orthonormal bases of linear subspaces generated by:
a) $L_1 = \{(0, 3, 4, 0)^T, (0, 0, 5, 0)^T, (2, 1, 0, 2)^T\}$
b) $L_2 = \{(2, 0, 1, 2)^T, (4, 3, 2, 4)^T, (6, -5, 3, 6)^T, (-4, 2, 4, 2)^T\}$

6: (*Applications of* $(A - G)$. ***D14 only***)
Solve the following classical calculus problems by making use of $(A - G)$ inequality.
a) Find the largest circular cylinder that can be inscribed in a sphere of a given radius.
b) Find the smallest radius r such that a circular cylinder of volume 8 cubic units can be inscribed in the sphere of radius r.