Math-484 Homework #8 (KKT and duality)

I will finish the homework before 10am Oct 31. If I spot a mathematical mistake I will let the lecturer know as soon as possible.

I will write clearly and neatly as the grader is not an expert in cryptography. I will sign each paper of my work and indicate if I am C14 (4 hours student).

1: (*Do I understand linear programming?*)

You have $12,000 to invest, and three different funds from which to choose. The municipal bond fund has a 7% return, the local bank's CDs have an 8% return, and the high-risk account has an expected (hoped-for) 12% return. To minimize risk, you decide not to invest any more than $2,000 in the high-risk account. For tax reasons, you need to invest at least three times as much in the municipal bonds as in the bank CDs. Assuming the year-end yields are as expected, what are the optimal investment amounts?

Formulate the question using linear programming and solve the program by examining the set of the feasible solutions. Invest all money you have. Use only two variables - make the problem 2D.

2: (*Can I use KKT?*)

Apply the Karush-Kuhn-Tucker Theorem to locate all solutions of the following convex programs:

$$(P_a)\begin{cases}\text{Minimize} & f(x_1,x_2)=e^{-(x_1+x_2)}\\ \text{subject to} & e^{x_1}+e^{x_2}\le 20\\ & x_1\ge 0\end{cases}\qquad (P_b)\begin{cases}\text{Minimize} & f(x_1,x_2)=x_1^2+x_2^2-4x_1-4x_2\\ \text{subject to} & x_1^2-x_2\le 0\\ & x_1+x_2\le 2\end{cases}$$

3: (*Can I solve geometric program?*)

Consider the following geometric program:

$$(GP)\begin{cases}\text{Minimize} & f(t_1,t_2)=t_1^{-1}t_2^{-1}\\ \text{subject to} & \frac{1}{2}t_1+\frac{1}{2}t_2\le 1\\ \text{where} & t_1>0,\ t_2>0\end{cases}$$

a) Convert (GP) to an equivalent convex program and solve the resulting program using KKT.

b) Solve the given (GP) by using methods of Chapter 5.3.

4: (*More geometric programming.*)

Solve the following geometric program:

$$(GP)\begin{cases}\text{Minimize} & x^{1/2}+y^{-2}z^{-1}\\ \text{subject to} & x^{-1}y^2+x^{-1}z^2\le 1\\ \text{where} & x>0,\ y>0,\ z>0\end{cases}$$

5: (*Can I solve simple things?*)
Let $f(x)$ be a differentiable function on $\mathbb{R}$. Suppose x_0 is fixed and there exists a number $\alpha \in \mathbb{R}$ such that

$$f(x) \geq f(x_0) + \alpha(x - x_0)$$

for all $x \in \mathbb{R}$. Show that $\alpha = f'(x_0)$.

6: (*C14 only*)
Let M be a subspace of $\mathbb{R}^n$. From the definitions, it is clear that $M \subseteq (M^{\perp})^{\perp}$. Use the Basic Separation Theorem to show $(M^{\perp})^{\perp} \subseteq M$. Thus giving a proof that $M = (M^{\perp})^{\perp}$.