Math-484 Homework #10 (Penatly methods and iterative methods)

I will finish the homework before 10am Dec 5. If I spot a mathematical mistake I will let the lecturer know as soon as possible.

I will write clearly and neatly as the grader is not an expert in cryptography. I will sign each paper of my work and indicate if I am C14 (4 hours student).

1: (*Page 235, exercise 1*)

Consider the following program:

$$(P) \begin{cases} \text{Minimize} & f(x) = x^2 - 2x \\ \text{subject to} & 0 \leq x \leq 1. \end{cases}$$

(a) Sketch the graphs of the Absolute Value and Courant-Beltrami Penalty Terms for (P).

(b) For each positive integer k, compute minimizer x_k of the corresponding unconstrained objective function $P_k(x)$ with the Courant-Beltrami Penalty Term.

(c) For each positive integer k, compute the minimizer x_k of the corresponding unconstrained objective function $F_k(x)$ with the Absolute Value Penalty Term.

2: (*Page 236, exercise 2*)

a) Use the penalty function method with the Courant-Beltrami penalty term to solve the problem (P).

$$(P) \begin{cases} \text{Minimize} & f(x_1, x_2) = x_1 + x_2 \\ \text{subject to} & x_1^2 - x_2 \leq 2 \end{cases}$$

b) Show that the objective function $F_k(\mathbf{x})$ corresponding to the Absolute value penalty term has no critical points off the parabola

$$x_1^2 - x_2 = 2$$

for $k > 1$ and compute the minimizer of $F_k(\mathbf{x})$.

3: (*Page 128, exercise 2*)

Show that the function $f(x) : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = x^{4/3}$$

has a unique global minimizer at $x^* = 0$ but that, for any nonzero initial point x_0, the Newton's Method sequence $\{x_k\}$ with initial point x_0 for minimizing $f(x)$ diverges.

4: (*Page 129, exercise 3*)

(a) Compute quadratic approximation $q(\mathbf{x})$ for

$$f(x_1, x_2) = 8x_1^2 + 8x_2^2 - x_1^4 - x_2^4 - 1$$

at the point $(\frac{1}{2}, \frac{1}{2})$.
(b) Compute the minimum $\mathbf{x}^*$ of the quadratic approximation $q(\mathbf{x})$ at $(\frac{1}{2}, \frac{1}{2})$.

5: (*Page 129, exercise 4*)
Compute the first two terms $\mathbf{x}_1, \mathbf{x}_2$ of the Newton's Method sequence $\{\mathbf{x}_k\}$ for minimizing the function

$$f(x_1, x_2) = 2x_1^4 + x_2^2 - 4x_1x_2 + 5x_2$$

with initial point $\mathbf{x}_0 = (0, 0)$.

6: (*Page 130, exercise 8*)
Compute the first two terms $\mathbf{x}_1, \mathbf{x}_2$ of the Steepest Descent sequence $\{\mathbf{x}_k\}$ for minimizing the function

$$f(x_1, x_2) = 2x_1^4 + x_2^2 - 4x_1x_2 + 5x_2$$

with initial point $\mathbf{x}_0 = (0, 0)$.

7: (*Page 131, exercise 14*)
Compute the first two iterates $\mathbf{x}_1, \mathbf{x}_2$ using the Broyden's Method for minimizing the function

$$f(x_1, x_2) = 2x_1^2 + x_2^2 - x_1x_2$$

with initial point $\mathbf{x}_0 = (1, 4)$.and with
(a) $D_0 = I$
(b) $D_0 = Hf(\mathbf{x}_0)$

8: (*Page 132, exercise 20*)
Compute the first two terms of the BFGS sequence and the first two terms of the DFP sequence for minimizing the function

$$f(x_1, x_2) = x_1^2 - x_1x_2 + \frac{3}{2}x_2^2$$

starting with the initial point $\mathbf{x}_0 = (1, 2)$ and $D_0 = I$. For each case, choose $t_k > 0$ to be the exact minimizer of $f(\mathbf{x})$ in the search direction from $\mathbf{x}_k$

9: (***C14 only***)
Prove that the DFP updates D_k are positive definite under the same hypotheses as Theorem 3.5.2 for the BFGS update.