Math-589 Homework #3

If I spot a mistake I will let the lecturer know as soon as possible. I will type the solution and send it as a PDF if I want to get a grade.
Alternatively, I can write two pages about my research that is using non-linear optimization.

1: (*Steepest descent*)
Describe general step of Steepest descent method.
Compute one step of the method on function

$$f(x, y) = 2x^2 - 4xy + 4y^2$$

with initial point $\mathbf{x}_0 = (2, 3)$.

2: (*DFP and BFGS*)
Describe general step of BFGS method and DFP method.
Pick BFGS or DFP and compute $\mathbf{x}_2$ of the minimizing sequence for the function

$$f(x_1, x_2) = x_1^2 - x_1 x_2 + \frac{3}{2} x_2^2$$

where the initial point $\mathbf{x}_0 = (1, 2)$ and $D_0 = I$. Suppose that $t_1 = 1$ and that $\mathbf{x}_1 = (1, \frac{1}{3})$.

3: (*Describing a set using semidefinite program*)
Show that set M can be described by a semidefinite program
1) $M = \{(x, y) \in (\mathbb{R}^+)^2 : \quad x \geq 1/y\}$
2) $M = \{(x, y) \in \mathbb{R}^2 : \quad x^4 + y^4 \leq 1\}$
Note that the desired program (P) can have more variables than just x and y. The set M is then taken as a projection of the set of feasible solutions of (P) to (x, y).

4: (*Functions of graphs*)
Let $G = ([n], E)$ be a graph where $[n] = \{1, \ldots, n\}$ and let $\alpha_{i,j} \leq \beta_{i,j}$ for every $\{i, j\} \in E$ be real numbers. Show that deciding if there exist $\mathbf{p}_1, \ldots, \mathbf{p}_n \in \mathbb{R}^n$ such that

$$\alpha_{i,j} \leq \|\mathbf{p}_i - \mathbf{p}_j\|_2 \leq \beta_{i,j}, \quad \{i, j\} \in E$$

can be formulated as a question if some semidefinite program is feasible.

5: (*Approximation algorithm*)
Find a probabilistic polynomial 0.878-approximation algorithm for the following problem:

MAX-2-SAT: Given a 2-CNF formula in n variables, determine the maximal number of clauses that can be simultaneously satisfied.

Note: Let X be a set of variables. A *literal* is any variable or its negation, a *k-clause* is a disjunction of k literals, and a *k-CNF formula* is a conjunction of k-clauses. A clause is satisfied by an assignment $f : X \to \{0, 1\}$ if it evaluates to 1 (true) under the assignment f.
For more detailes about CNF see
`http://en.wikipedia.org/wiki/Conjunctive_normal_form`.