MATH413 HW 5

due **Feb 27** before class, answer without justification will receive 0 points. Staple all your papers.

1: *(P. 156, #29)* Find and prove a formula for

$$\sum_{r,s,t\geq 0 \text{ and } r+s+t=n} \binom{m_1}{r}\binom{m_2}{s}\binom{m_3}{t}$$

where the summation extends over all nonnegative integers r, s and t with sum $r + s + t = n$.

2: *(P. 158, #37)* Use the multinomial theorem to show that, for positive integers n and t,

$$t^n = \sum \binom{n}{n_1\ n_2\ \cdots\ n_t},$$

where the summation extends over all nonnegative integral solutions $n_1, n_2, \ldots, n_t$ of $n_1 + n_2 + \ldots + n_t = n$.

3: Find
(a) the coefficient of x^3y^7 in the expansion of $(2x + y)^{10}$;
(b) the coefficient of $x^{13}y^{77}$ in the expansion of $(3x - 2y)^{90}$;
(c) the coefficient of $x_1^3x_2^3x_3x_4^2$ in the expansion of $(x_1 - x_2 + 2x_3 - 2x_4)^9$

4: *(P. 158, #44)* Prove that,

$$\sum_{n_1+n_2+n_3=n} \binom{n}{n_1\ n_2\ n_3}(-1)^{n_1-n_2+n_3} = (-3)^n$$

where the summation extends over all nonnegative integral solutions of $n_1 + n_2 + n_3 = n$.

5: *(P. 160, #50)* Consider the partially ordered set (X, R), where $X = \{1, 2, \ldots, 12\}$ and xRy if y is divisible by x, i.e., $R = \{(x, y) \mid \ y \text{ is divisible by } x\ \}$.
(a) Determine a chain of largest size and a partition of X into the smallest number of antichains.

(b) Then determine an antichain of largest size and a partition of X into the smallest number of chains.

6: *(P. 198, #2)* Find the number of integers between 1 and $10,000$ inclusive that are not divisible by 4, 6, 7, or 10.

7: *(P. 198, # 6)* A bakery sells chocolate, cinnamon, and plain doughnuts and at a particular time has 6 chocolate, 6 cinnamon, and 3 plain. If a box contains 12 doghnuts how many different options are there for a box of dougnuts?

8: *(P. 198, # 8.)* Determine the number of solutions of the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 14$ in positive integers x_1, x_2, x_3, x_4 and x_5 not exceeding 5.