MATH413 HW 6

due **Mar 6** before class, answer without justification will receive 0 points. Staple all your papers.

1: *(P. 199, #13)* Determine the number of permutations of {1,2,...,9} in which at least one odd integer is in its natural position.

2: *(P. 199, #17)* Determine the number of permutations of the multiset

$$S = \{3 \cdot a, 4 \cdot b, 2 \cdot c\},$$

where, for each type of letter, the letters of the same type do not appear consecutively. (Thus *abbbbcaca* is not allowed but *abbbacacb* is.)

3: *(P.200, #21)* Prove that D_n is an even number if and only if n is an odd number.

4: *(P.200, #22)* Show that the numbers Q_n of Section 6.5 can be rewritten in the form

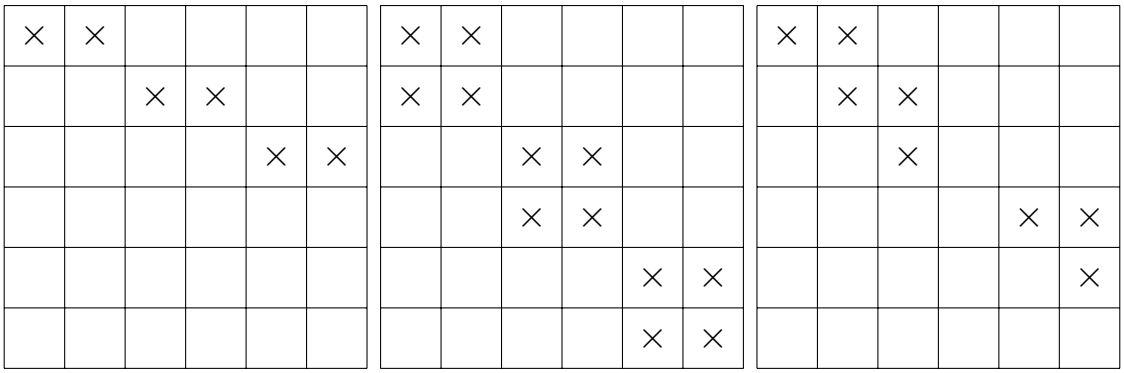
$$Q_n = (n-1)!\left(n - \frac{n-1}{1!} + \frac{n-2}{2!} - \frac{n-3}{3!} + \cdots + \frac{(-1)^{n-1}}{(n-1)!}\right)$$

5: *(P.200, #23)* Use the identity

$$(-1)^k \frac{n-k}{k!} = (-1)^k \frac{n}{k!} + (-1)^{k-1} \frac{1}{(k-1)!}$$

to prove that $Q_n = D_n + D_{n-1}, (n = 2, 3, \ldots)$.

6: *(P. 200, # 24)* What is the number of ways to place six nonattacking rooks on the 6-by-6 boards without forbidden positions as shown?



(a) (b) (c)

7: *(P. 201, # 27.)* A carousel has eight seats, each representing a different animal. Eight girls are seated on the carousel facing forward (each girl looks at another girl's back). In how many was can the girls change seats if that each has a different girl in from of her. How does the problem change if all the seats are identical?

8: *(P. 200, #32)* Let n be a positive integer and let $p_1, p_2, \ldots, p_k$ be all the different prime numbers that divide n. Consider the Eurler function ϕ defined by

$$\phi(n) = |\{k : 1 \leq k \leq n, \text{GCD}\{k, n\} = 1\}|.$$

Use the inclusion-exclusion principle to show that

$$\phi(n) = n \prod_{i=1}^{k} (1 - \frac{1}{p_i}).$$