MATH413 HW 8

due **Apr 10** before class, answer without justification will receive 0 points. Staple all your papers.

1: *P.260, #22* Determine the exponential generating function for the sequence of factorials

$$0!, 1!, 2!, 3!, \ldots, n!, \ldots$$

2: *P.260, # 24* Let S denote the multiset $\{\infty \cdot e_1, \infty \cdot e_2, \ldots, \infty \cdot e_k\}$. Determine the exponential generating function for the sequence $h_0, h_1, h_2, \ldots, h_n, \ldots$, where $h_0 = 1$ and for $n \geq 1$,
(b) h_n equals the number of n-permutations of S in which each object occurs at least four times.
(c) h_n equals the number of n-permutations of S in which e_1 occurs at least once, e_2 occurs at least twice, ..., e_k occurs at least k times.

3: *P.260, #23* Let α be a real number. Let the sequence $h_0, h_1, h_2, \ldots, h_n, \ldots$ be defined by $h_0 = 1$, and $h_n = \alpha(\alpha - 1) \cdots (\alpha - n + 1), (n \geq 1)$. Determine the exponential generating function for the sequence.

4: *P. 260, #26* Determine the number of ways to color squares of a 1-by-n chessboard using the colors red, blue, green, and orange if an even number of squares is to be colored red and an even number is to be colored green.

5: *P. 261, #32* Solve the recurrence relation $h_n = (n + 2)h_{n-1},\ (n \geq 1)$ with initial value $h_0 = 2$.

6: *P. 261, #34* Solve the recurrence relation $h_n = 8h_{n-1} - 16h_{n-2},\ (n \geq 2)$ with initial values $h_0 = -1$ and $h_1 = 0$.

7: Determine the generating function for the sequence $\{h_n\}_{n=0}^{\infty}$ that satisfies the relation $h_n = 6h_{n-1} - 8h_{n-2}$ for $n \geq 2$ with initial conditions $h_0 = 1, h_1 = 0$. Using the generating function find an explicit formula for h_n in this problem.

8: *P. 262 #40* Let a_n equal the number of ternary strings of length n made up of 0s, 1s, and 2s, such that the substrings 00, 01, 10, and 11 never occur. Prove that

$$a_n = a_{n-1} + 2a_{n-2}, \ (n \geq 2),$$

with $a_0 = 1$ and $a_1 = 3$. Then find a formula for a_n.