MATH413 HW 9

due **Apr 17** before class, answer without justification will receive 0 points. Staple all your papers.

1: Solve (find expression for h_n) the following recurrences:
(a) $h_{-1} = 3, h_0 = 4, h_{n+1} = 4h_n - 3h_{n-1}$
(b) $h_0 = 3, h_1 = 4, h_{n+2} = 4h_n + 2$

2: *(P.262,#42)* Solve the nonhomogeneous recurrence relation

$$h_n = 4h_{n-1} + 4^n, \ (n \geq 1)$$
$$h_0 = 3.$$

3: *(P.263,#47)* Solve the nonhomogeneous recurrence relation

$$h_n = 4h_{n-1} - 4h_{n-2} + 3n + 1$$
$$h_0 = 1$$
$$h_1 = 2.$$

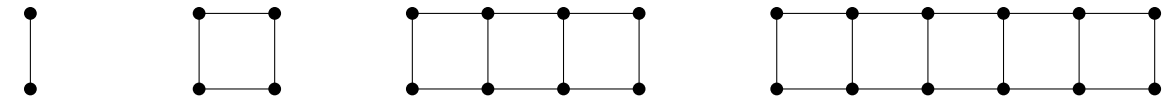
4: *(P.264,#51)* Solve the recurrence relation

$$h_n = 3h_{n-1} + 4^n, \ (n \geq 1)$$
$$h_0 = 2$$

using generating functions.

5: *(P.264,#53)* Suppose you deposit \$500 in a bank account -that pays 6% interest at the end of each year (compounded annually). Thereafter, at the beginning of each year you deposit \$100. Let h_n be the amount in your account after n years (so $h_0 = \$500$). Determine the generating function $g(x) = h_0 + h_1 x + \cdots + h_n x^n + \cdots$ and then a formula for h_n.

6: Count the number of perfect matchings of points of an earthworm W_{2n} on $2n$ vertices when only segments like in the picture may be used in the matching.



W_2 W_4 W_8 W_{12}

Recall that matching means that some edges are deleted and in the result every vertex (black dot) has exactly one incident edge left. For example W_4 has two perfect matchings (a) and (b):

(a) (b)