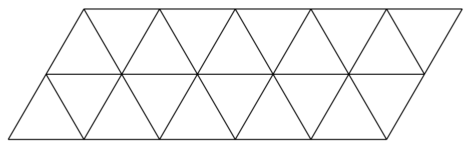
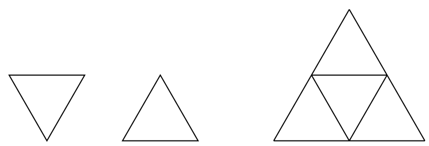
MATH413 HW 10

due **Apr 24** before class, answer without justification will receive 0 points. Staple all your papers.

1: Find the number of possible tilings of triangular piece $n \times 2$. Example for $n = 5$.



using the following three kinds of pieces:

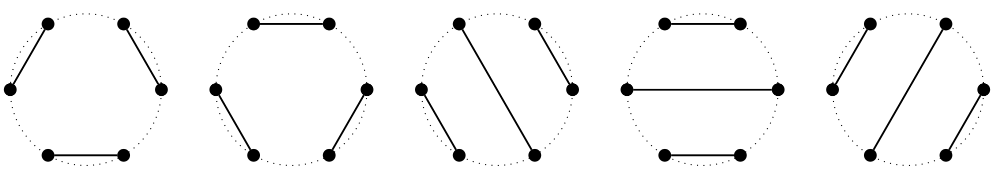


Another way – suppose that you can cut the tripe of triangles along lines. After the cutting, you are left with pieces that look like a triangle rotated by 180degrees, triangle or a piece that is a composition of four triangles. The pieces are like this up to translation (no rotation allowed). How many different cuttings are there?

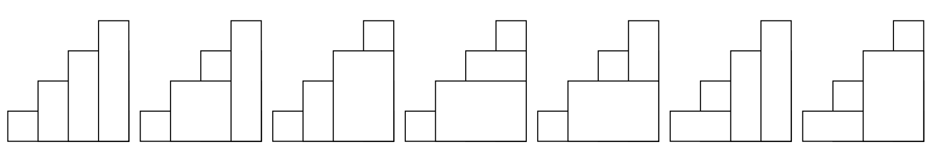
2: Let there be $2n$ points V on a circle in the plane. A *perfect matching* M is a set of segments with endpoints only from V and every point in V is an endpoint of exactly one segment. Note that $|M| = n$ as one segment needs exactly 2 points from V. A matching M in *non-crossing* if the segments are disjoint. Find the number of non-crossing perfect matchings for $2n$ points.

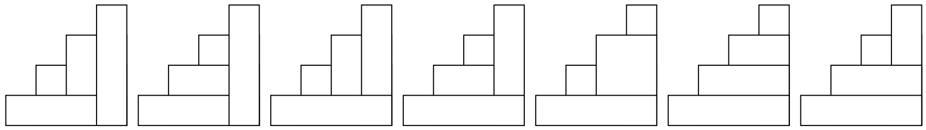
This can be stated in graph theory language as follows. Count the number of perfect matchings of K_{2n} with vertices are vertices of a regular $2n$-gon in the plane such that the edges of the matching do not cross.

Example for $n = 3$ and hence 6 points.



3: Find the number of possibilities to build stairs of height n using n rectangular bricks. All the possibilities for $n = 4$ are depicted.





4: *(P. 315, #2)* Prove that the number of 2-by-n arrays

$$\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \end{bmatrix}$$

that can be made from numbers $1, 2, \ldots, 2n$ such that

$$x_{11} < x_{12} < \cdots < x_{1n}$$

$$x_{21} < x_{22} < \cdots < x_{2n}$$

$$x_{11} < x_{21}, x_{12} < x_{22}, \ldots, x_{1n} < x_{2n},$$

equals the n^{th} Catalan number, C_n.

5: Using the difference sequence method, find a closed form the following sum:

$$\sum_{k=0}^{n} k^4 - k.$$

6: *(P.316, #7)* The general term h_n of a sequence is a polynomial in n of degree 3. If the first four entries in the 0^{th} row of its difference table are 1,-1,3,10, determine h_n and a formula for $\sum_{k=0}^{n} h_k$.

7: *(P.316, #8)* Find the sum of the fifth powers of the first n positive integers.