Math-484 List of definitions and theorems

This is a list of definitions that a student of 484 is required to know.

Definitions (Midterm 1):

- cosine of two vectors *page 6*
- distance of two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$
- ball $B(\mathbf{x}, r)$ (what is $\mathbf{x}$ and r?) *page 6*
- interior D^0 of set $D \subseteq R^n$ *page 6, page 164*
- open set $D \subseteq \mathbb{R}^n$ *page 6*
- closed set $D \subseteq \mathbb{R}^n$ *page 7*
- compact set $D \subseteq \mathbb{R}^n$ *page 6*
- (global,local)(strict)minimizer and maximizer of a function $f : \mathbb{R}^n \to \mathbb{R}$ *page 8*
- critical point of a function $f : \mathbb{R}^n \to \mathbb{R}$ *page 8*
- gradient $\nabla f(\mathbf{x})$ where $f : \mathbb{R}^n \to \mathbb{R}$ *page 10*
- Hessian $Hf(\mathbf{x})$ where $f : \mathbb{R}^n \to \mathbb{R}$ *page 10*
- quadratic form associated with a symmetric matrix A *page 12*
- (positive,negative)(semi)definite matrix *page 13*
- indefinite matrix *page 13*
- saddle point of a function $f : \mathbb{R}^n \to \mathbb{R}$
- Δ_k, the k^{th} principal minor of a matrix A *page 16*
- $f : \mathbb{R}^n \to \mathbb{R}$ being coercive *page 25*
- eigenvalues and eigenvectors of a matrix A *page 29*
- $C \subseteq \mathbb{R}^n$ being convex *page 38*
- closed and open half-spaces in $\mathbb{R}^n$ *page 40*
- convex combination of k vectors from $\mathbb{R}^n$ *page 41*
- convex hull of $D \subseteq \mathbb{R}^n$ *page 42*
- (strictly) convex and concave function $f : C \to \mathbb{R}$, where $C \subseteq \mathbb{R}^n$ *page 49*

Theorems and statements (for Midterm 1):
(*Try to do not ignore assumptions - like that sometimes the function must be continuous etc.*)
(*Proofs are only for students with 4-credits*)
- State State Cauchy-Swartz inequality (page 6)
- What can you tell about minimizers and maximizers of continuous $f : I \to \mathbb{R}$ where $I \subset \mathbb{R}$ is a closed interval? *(Theorem 1.1.4)*
- Which minimizers or maximizers of $f : D \to \mathbb{R}$ must be critical points? *(Theorem 1.2.3?)*
- State the cornerstone theorem for using gradient and Hessian for finding minimizers. *(Theorem 1.2.4)*
- How can H_f help identify global minimizers and maximizers? *(Theorem 1.2.5 or 1.2.9)*
- How principal minors of matrix A correspond to positive(negative) (semi)definity or indefinity of A? *(Theorem 1.3.3)*
- How can H_f help identify local minimizers and maximizers? *(Theorem 1.3.6, **with proof**)*
- Do coercive functions have some special properties related to minimizers? *(Theorem 1.4.4, **with proof**)*
- How eigenvalues of a symmetric matrix A correspond to positive/negative (semi)definity of A? *(Theorem 1.5.1)*
- Is there any connection between the convex hull of D, $co(D)$, and set of all convex combinations of vectors from D? ($D \subseteq \mathbb{R}^n$) *(Theorem 2.1.4)*
- Is there a convex function $\mathbb{R} \to \mathbb{R}$ that is not continuous? *(Theorem 2.3.1)*
- Do (local) minimizers of a convex function have some nice properties? *(Theorem 2.3.4 **with proof**)*
- Do (local) maximizers of a concave function have some nice properties? *(Theorem 2.3.4)*
- Can be a convex function recognized by its gradient? *(Theorem 2.3.5)*
- Do critical points of convex functions have some nice properties? *(Theorem 2.3.5 + Corollary)*
- What is the correspondence between Hessian and convexity of a function? (in $\mathbb{R}^n$) *(Theorem 2.3.7)*
- Is it possible to decide if a function is convex by decomposing it to simpler ones? How? *(Theorem 2.3.10)*
- State A-G inequality and when it is equality *(Theorem 2.4.1 **with proof**)*

Definitions (Midterm 2):

- posinomial *page 67*
- unconstrained geometric program
- primal and dual geometric program *page 67,68*
- feasible solution of a program (P)
- hyperplane H in $\mathbb{R}^n$ *page 158*
- boundary point of $C \subset \mathbb{R}^n$ *page 158*
- closure $\overline{A}$ of $A \subset \mathbb{R}^n$ *page 163*
- subgradient of $f : \mathbb{R}^n \to \mathbb{R}$ *page 168*
- subdifferential of $f : \mathbb{R}^n \to \mathbb{R}$ *page 168*
- general form of a convex program (P) *page 169*
- feasible vector (or feasible solution) of a program (P) *page 169*
- feasible region of a program (P) *page 169*
- consistent program (P) *page 169*
- superconsistent program (P) *page 169*
- MP for program (P) - also define (P) *page 171*
- $MP(z)$ for program $(P(z))$ - also define $(P(z))$ *page 171*
- sensitivity vector of a program (P) *page 177*
- Lagrangian $L(\mathbf{x}, \lambda)$ of a program (P)*page 182*
- complementary slackness conditions for a program (P) *page 184*
- general form of constrained geometric program (GP) and its dual (DGP) *page 193*

Theorems and statements (Midterm 2):

- Describe transition form unconstrained geometric program to its dual using A-G inequality *(pages 67, 68)*.
- What is the way of computing of the closest vector of a convex set to a given vector? *Theorem 5.1.1*
- What is a sufficient condition for existence of a closest vector from a set C to a given vector $\mathbf{x}$? *Theorem 5.1.3*
- What is a sufficient condition for existence of a unique closest vector from a set C to a given vector $\mathbf{x}$? *Corollary 5.1.4*
- State basic separation theorem. *Theorem 5.1.5, **with proof***
- State Support theorem. *Theorem 5.1.9*
- What can you say about $MP(z)$ if (P) is super consistent? *Theorem 5.2.6*
- Are there sufficient conditions for convex program (P) to have a sensitivity vector? *Theorem 5.2.8, **with proof***
- Can MP be computed from the sensitivity vector? *(Theorem 5.2.11), **with proof***
- State Karush-Kuhn-Tucker Theorem (Saddle point version) *Theorem 5.2.13*
- State Karush-Kuhn-Tucker Theorem (Gradient form) *Theorem 5.2.14*
- State Extended Arithmetic-Geometric Mean Inequality Include also when it is equality! *Theorem 5.3.1, **with proof***
- What are sufficient condition for a constrained geometric program (GP) to have no duality gap? *Theorem 5.3.5*

Definitions (Midterm 3):

- dual of a convex program *page 200*
- duality gap *page 209*
- absolute value penalty function *page 217*
- Courant-Beltrami penalty function *page 219*
- generalized penalty function *page 223*
- Jacobian Matrix of a function $g : \mathbb{R}^n \to \mathbb{R}^n$ *page 85*
- describe Newton's method for function minimization *page 88, 3.1.3*
- describe Steepest descent method *page 98, 3.2.1*
- Descent method *page 103*
- secant condition *page 114*
- outer product or tensor product *page 115*
- describe Broyden's method *page 117, 3.4.1*
- distance between two matrices *page 118, 3.4.3*
- describe BFGS method *page 125, 3.5.3*
- describe DFP method *page 127, 3.5.4*

Theorems and statements (Midterm 3):

- State the strong duality theorem for linear programming. *page 203*
- State duality theorem for convex programming. *Theorem 5.4.6*
- State the theorem that gives properties of Courant-Beltrami penalty function. *Theorem 6.2.3*
- What is the effect of the coercive objective function on the duality? *Theorem 6.3.1* (With proof)
- For a convex program (P), what can you tell about (P^{ε}) and MP^{ε}? *Theorem 6.3.2*
- When Newton's method converges in one step? *Theorem 3.1.4*
- When is Newton's method guaranteed to do decreasing steps? *Theorem 3.1.5*
- What is the special property of the steps in Steepest descent method? *Theorem 3.2.3*
- When is the Steepest descent method really a descent method? *Theorem 3.2.5*
- What is a sufficient condition for the Steepest method to converge? *Theorem 3.2.6*
- State the conditions that a good descent method should satisfy. Write them formally as well as simple explanation in English. *(page 106,107), 4 credits also why they do that they do*
- State Wolfe's Theorem about existence about descent methods. *Theorem 3.3.1*
- Describe modification of Newton's Method such that it can be used with Wolfe's Theorem. *page 111*
- What distance property is satisfied by D_{k+1} in the Boroyden's Method? *Theorem 3.4.5*
- If two vectors $\mathbf{a}, \mathbf{b}$ have $\mathbf{a}^T\mathbf{b} > 0$, can you tell something about mapping $\mathbf{a}$ to $\mathbf{b}$ using a matrix? *Theorem 3.5.1*

Definition (SDP) and Interior Point Method

- Trace of a matrix A
- dot product for two matrices A and B
- A general form of (SDP)
- Dual semidefinite program $(DSDP)$
- Strictly feasible (SDP) and $(DSDP)$
- Write a convex program (P) and a barrier function corresponding to it.

Definition (SDP) and Interior Point Method

- State duality theorem for Semidefinite programming
- State theorem about efficiently solving (SDP)

Other questions about stuff

- Is it true that a strictly convex function has a global minimizer? Why?
- Let x be a critical point of f and $Hf(x)$ be positive semidefinite. Is x local minimizer? Why?
- Let $f(x) = f_1(x) \cdot f_2(x)$ where both f_1 and f_2 are convex. Is f convex? Why?
- Let f be a (not strictly) concave function. Is it true that if x is a critical point and $Hf(x)$ is negative definite, then x is a local maximizer? Why?
- Is it true that every two convex sets $C, D \subset \mathbb{R}^n$ can be strictly separated? That is, there exists $\mathbf{a} \in \mathbb{R}$ for every $\mathbf{c} \in C$ and $\mathbf{d} \in D$

$$\mathbf{a}^T\mathbf{c} < \mathbf{a}^T\mathbf{d}$$

- What is the relation of sensitivity vector of (P) and λ from KKT conditions?
- Is $MP(z)$ convex, differentiable or continuous?
- What can you tell about a program (P) if you know its sensitivity vector λ?
- How to derive dual of a geometric program using extended AG inequality?
- Derive a dual convex program from convex program. *page 200*
- Is it true that for every convex program its optimum value is equal to the optimum value of the dual?
- Describe penalty function method
- What are differences in behavior of the Absolute value penalty function and Courant-Beltrami penalty function?
- How to modify any convex function to a coercive one? (why it is coercive?)
- What are relations between $MP, MP^\epsilon, MD, MD^\epsilon$?
- By what is Newton's method approximating the function for minimization?
- Give derivation of the update matrix for D_k in Broyden's method. *page 115,116*
- Give derivation of the update matrix for D_k in BFGS method. *page 124*
- Is it true the every semidefinite program is efficiently solvable? (in polynomial time)

Extra

- express given linear program as a semidefinite program
- express given program with quadratic constraints as a semidefinite program