Math-484 Homework #5 (($A - G$) inequality, geometric programming)

I will finish the homework before 11am Oct 2. If I spot a mathematical mistake I will let the lecturer know as soon as possible.
I will write clearly and neatly as the grader is not an expert in cryptography. I will sign each paper of my work and indicate if I a 4 credits student.

1: (*I will recall convexity of a function (how we proved (A-G) inequality)*)
Show that for all positive x and y:

$$\frac{x}{4} + \frac{3y}{4} \leq \sqrt{\ln\left(\frac{e^{x^2}}{4} + \frac{3}{4}e^{y^2}\right)}$$

Hint: The desired inequality follows from convexity of an appropriate function.

2: (*Applications of* $(A - G)$)
Solve the following classical calculus problems by making use of $(A - G)$ inequality.
a) Find the largest circular cylinder that can be inscribed in a sphere of a given radius.
b) Find the smallest radius r such that a circular cylinder of volume 8 cubic units can be inscribed in the sphere of radius r.

3: (*I want to know* (GP))
State the dual (DGP) of the following (GP) and solve the (GP) using (DGP). Solving means, finding optimal $\mathbf{x}^* = (x_1, x_2)$ and value of the objective function.

$$(GP)\begin{cases}\text{Minimize} & (5^4)\frac{x_2^2}{x_1} + \frac{x_3}{5x_1x_2^2} + \frac{25x_1}{2} + \frac{1}{10x_1x_3^2} \\ \text{subject to} & x_1, x_2, x_3 > 0\end{cases}$$

After solving (GP) by hand, input the program to `http://www.wolframalpha.com` to check your solution and an enclose both your manual solution and prinout of the Wolfram solution to your homework.

4: (*I wanna be a* (GP) *master!* ***C14 only***)
Solve the following (GP) where c_1, c_2, c_3 are positive numbers:

$$(GP)\begin{cases}\text{Minimize} & f(x,y) = c_1x + c_2x^{-2}y^{-3} + c_3y^4 \\ \text{subject to} & x, y > 0\end{cases}$$

Hint: (The result is not particularly nice.)