Math-484 Homework #12 (iterative methods II)

*I will finish the homework before 11am Dec 2 (**Monday**). If I spot a mathematical mistake I will let the lecturer know as soon as possible.*

I will write clearly and neatly as the grader is not an expert in cryptography. I will sign each paper of my work and indicate if I am a 4 credits student.

1: (*If* $(Hf)^{-1}$ *correct to use in iterative methods?*)

Prove that if A is a positive definite matrix, then A^{-1} exists and in positive definite.

(Recall that A is also symmetric since it is part of being positive definite.)

2: (*Making matrices positive definite*)

(a) Show that if A is a symmetric matrix, then

$$\lambda = \max\{\mathbf{x}^T A\mathbf{x} : \mathbf{x} \in \mathbb{R}^n, \|x\| = 1\}$$

where λ is the largest eigenvalue of A.

(Hint: Diagonalize A with an orthogonal matrix.)

(b) Show that

$$\max\{|\mathbf{x}^T A\mathbf{x}| : \mathbf{x} \in \mathbb{R}^n, \|x\| = 1\}$$

is equal to the absolute value of the eigenvalue of A with largest absolute value.

(c) For each of the following symmetric matrices A find a positive numer μ such that $A+\mu I$ is positive definite:

$$(i)\ A = \begin{pmatrix} 3 & 7 & 0 \\ 7 & 5 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (ii)\ A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{pmatrix}.$$

3: (*Try Broyden's method*)

Compute the first two iterates $\mathbf{x}_1, \mathbf{x}_2$ using the Broyden's Method for minimizing the function

$$f(x_1, x_2) = 2x_1^2 + x_2^2 - x_1x_2$$

with initial point $\mathbf{x}_0 = (1, 4)$ and with

(a) $D_0 = I$

(b) $D_0 = Hf(\mathbf{x}_0)$

4: (*Try BFGS and DFP yourself*)

Compute the first two terms of the BFGS sequence and the first two terms of the DFP sequence for minimizing the function

$$f(x_1, x_2) = x_1^2 - x_1x_2 + \frac{3}{2}x_2^2$$

starting with the initial point $\mathbf{x}_0 = (1, 2)$ and $D_0 = I$. For each case, choose $t_k > 0$ to be the exact minimizer of $f(\mathbf{x})$ in the search direction from $\mathbf{x}_k$.

5: (***4 credits only***)
Prove that the DFP updates D_k are positive definite under the same hypotheses as Theorem 3.5.2 for the BFGS update.