MATH201 MIDTERM 2 - practice

1: Let f be a function from $\mathbb{R}$ to $\mathbb{R}$ and let b be a real number. For the following sentence, write it in symbolic logic, then negate it and write the negation as an English sentence.

For every positive number ε there is a positive number M for which $|f(x) - b| < \varepsilon$ whenever $x > M$.

2: Prove by contradiction: Suppose $a, b \in \mathbb{Z}$. If both ab and $a + b$ are even, then both a and b are even as well.

3: For real p, show that

$$p^3 + (1-p)^3 \geq \frac{1}{4}$$

holds with equality if and only if $p = \frac{1}{2}$.

4: Prove without using Venn diagrams that if A, B and C are sets, then $(A \cup B) - C = (A - C) \cup (B - C)$.

5: Prove or disprove: The inequality $2^x \geq x + 1$ is true for all positive real numbers x.

6: For any integer $n \geq 0$, it follows that $9|(4^{3n} + 8)$.

7: Concerning the Fibonacci sequence, prove that $F_1 + F_2 + F_3 + F_4 + \ldots + F_n = F_{n+2} - 1$.

8: Prove that for all $n \geq 1$ hods

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}.$$

9: Using mathematical induction show that evert set of size k has exactly 2^k different subsets.
Use induction on k.

10: Let F_n be a Fibonacci number. Prove that

$$F_{n+1}^2 - F_{n+1}F_n - F_n^2 = (-1)^n.$$

11: Prove that $\sqrt{5}$ is not a rational number.

12: Prove or disprove: There exist prime numbers p and q for which $p - q = 97$.