MATH304   HW 3

due **Sep 10** before class, **answer without justification will receive 0 points**. The solution has to be typed (using LaTeX).

1: *(P. 61, #11)* How many sets of three integers between 1 and 20 are possible if no two consecutive integers are to be in a set?

2:   There are $n \geq 4$ knights seated around King Arthur's round table and three of them are selected to be sent off to slay a troublesome dragon. What is the probability that at least two of the three were seated next to each other?

3: *(P. 65, #38.)* How many integral solutions of

$$x_1 + x_2 + x_3 + x_4 = 30$$

satisfy $x_1 \geq 2, x_2 \geq 0, x_3 \geq -5$, and $x_4 \geq 8$?
(Use substitution to get ≥ 0 for all variables.)

4:   How many integral solutions of

$$x_1 + x_2 + x_3 \leq 20$$

satisfy $x_1 > 3, x_2 \geq 0$ and $x_3 > -2$?

5:   For each $n > 0$, prove that there is an integer comprised only of the digits 0 and 1 that is divisible by n. (Hint: Pigeonhole principle)

6: *(P. 83, #11)* A student has 37 days to prepare for an examination. From past experience she knows that she will require no more than 60 hours of study. She also wishes to study at least 1 hour per day. Show that no matter how she schedules her study time (a whole number of hours per day, however), there is a succession of days during which she will have studied exactly 13 hours.