

MATH304 HW 5 - midterm tryout

due **Oct 1** before class, **answer without justification will receive 0 points**. The solution has to be typed (using L^AT_EX).

1: Prove that in a group of $n > 1$ people there are two who have the same number of acquaintances in the group. (It is assumed that no one is acquainted with oneself.)

2: Combinatorially prove the following binomial identity

$$\sum_{k=0}^r \binom{n+k}{k} = \binom{n+r+1}{r}.$$

3: There are 20 identical knights lined up in a row occupying 20 distinct places as follows:



Six of them will be replaced by rooks ♖. How many possible replacements are there if no two of the rooks can be next to each other?

4: Determine the number of ways to distribute 10 orange drinks, 1 lemon drink, and 1 strawberry drink to four thirsty students so that each student gets at least one drink, and the lemon and strawberry drinks go to different students.

5: How many five-digit integers n satisfy **all** of the following conditions:?
(first digit cannot be 0)

- (a) $n > 60000$.
- (b) the digits are distinct.
- (c) n is even.

6: There are $2n + 1$ identical books to be put in a bookcase with three distinguishable shelves. In how many ways can this be done if each pair of

shelves together contains more books than the other?

*(Only the number of books in each shelf gives a different way of putting.
Subtraction principle might help)*