MATH304   HW 6

due **Oct 8** before class, **answer without justification will receive 0 points**. The solution has to be typed (using LaTeX).

1: *(P. 155, #9)* Evaluate the sum

$$\sum_{k=0}^{n}(-1)^k\binom{n}{k}10^k.$$

2: *(P. 156, #13)* Find one binomial coefficient equal to the following expression:

$$\binom{n}{k}+3\binom{n}{k-1}+3\binom{n}{k-2}+\binom{n}{k-3}.$$

3: *(P. 156, #15)* Prove that for every integer $n > 1$,

$$\binom{n}{1}-2\binom{n}{2}+3\binom{n}{3}+\cdots(-1)^{n-1}n\binom{n}{n}=0.$$

4: *(P. 156, #20)* Find integers a,b, and c such that

$$m^3=a\binom{m}{3}+b\binom{m}{2}+c\binom{m}{1}$$

for all m. Then sum the series $1^3+2^3+3^3+\cdots+n^3$.

5: *(P. 157, #27)* Let n and k be positive integers. Give a combinatorial proof of the identity

$$n(n+1)2^{n-2}=\sum_{k=1}^{n}k^2\binom{n}{k}$$

6: *(P. 157, #24)* Consider a three-dimensional grid whose dimensions are 10 by 15 by 20. You are at the front lower left corner of the grid and you wish to get to the back top right corner 45 steps away. How many different routes are there that take exactly 45 steps?