MATH304 HW 7

due **Oct 15** before class, **answer without justification will receive 0 points**. The solution has to be typed (using $\LaTeX$).

1: Prove that the only antichain of $S = \{1, 2, 3, 4\}$ of size 6 is the antichain of all 2-subsets of S.

2: Let n and k be a positive integers. Give a combinatorial proof that

$$\sum_{k=1}^{n} k \binom{n}{k}^2 = n \binom{2n-1}{n-1}.$$

3: Let n be a positive integer. Prove that

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k}^2 = \begin{cases} 0 & \text{if } n \text{ is odd} \\ (-1)^m \binom{2m}{m} & \text{if } n = 2m. \end{cases}$$

Hint: consider $(1-x^2)^n = (1+x)^n(1-x)^n$.

4: Evaluate the sum

$$1 - \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} - \frac{1}{4}\binom{n}{3} + \cdots + (-1)^n \frac{1}{n+1}\binom{n}{n}.$$

5: Use **combinatorial** reasoning to prove the identity (in the given form)

$$\binom{n}{k} - \binom{n-3}{k} = \binom{n-1}{k-1} + \binom{n-2}{k-1} + \binom{n-3}{k-1}$$

6: Use Newton's binomial theorem to approximate $\sqrt{80}$.
(Hint: See page 148 and 149. First three digits after the decimal point is enough.)