MATH304 HW 9

due **Oct 29** before class, **answer without justification will receive 0 points**. The typing the HW in LaTeX is optional.

1: Let there be n seats around a round table. How many ways can you seat k people such that no two people sit next to each other. Seats are distinguishable, the circular shape is for defining neighboring seats. Assume $1 \leq k \leq \frac{n}{2}$, that is, the seating is indeed possible.
Your answer should be general for all n and k. Doing one concrete example is not a solution.

2: Let n be odd and $(x_1, x_2, \ldots, x_n)$ be a permutation of $[n]$. Show that $(x_1 - 1) \cdot (x_2 - 2) \cdots (x_n - n)$ is even.

3: Show that the numbers Q_n of Section 6.5 can be rewritten in the form

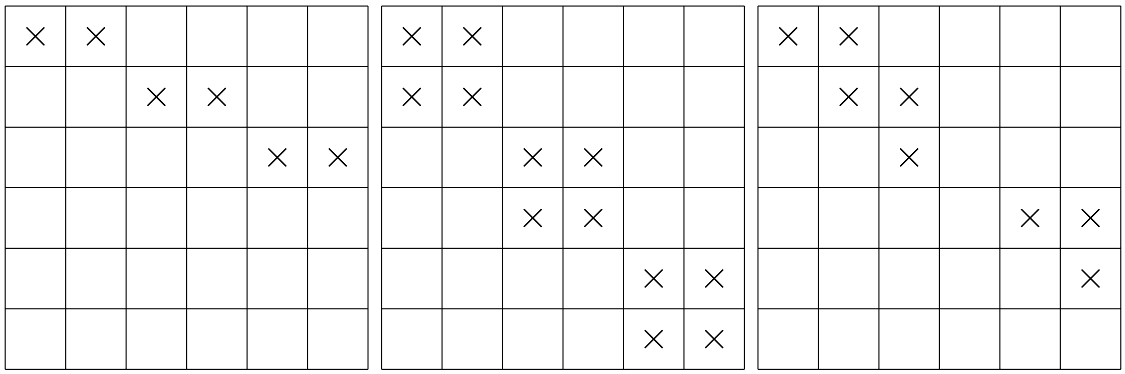
$$Q_n = (n-1)!\left(n - \frac{n-1}{1!} + \frac{n-2}{2!} - \frac{n-3}{3!} + \cdots + \frac{(-1)^{n-1}}{(n-1)!}\right)$$

4: Use the identity

$$(-1)^k \frac{n-k}{k!} = (-1)^k \frac{n}{k!} + (-1)^{k-1} \frac{1}{(k-1)!}$$

to prove that $Q_n = D_n + D_{n-1}, (n = 2, 3, \ldots)$.

5: What is the number of ways to place six nonattacking rooks on the 6-by-6 boards without forbidden positions as shown?



(a) (b) (c)

6: A carousel has eight seats, each representing a different animal. Eight boys are seated on the carousel but facing inward, so that each boy faces another (each boy looks at another boy's front). In how many ways can be the boys change seats so that each faces a different boy? How does the problem change if all seats are identical?

Example: Boys sitting opposite of each other are $P = \{\{1,5\},\{2,6\},\{3,7\},\{2,8\}\}$, depicted on the left. One of the seating we try to count is in the middle, since the opposite pairs are $\{\{1,2\},\{3,4\},\{5,7\},\{6,8\}\}$. On the right is a seating that we are NOT counting since the 1 and 5 are opposite of each other.