MATH304   HW 10

due **Nov 12** before class, **answer without justification will receive 0 points**. The typing the HW in LaTeX is optional.

1:   Use Fibonacci numbers to count the following problems:

(a) Count the number of subsets of $\{1, 2, \ldots, n\}$ that do not have having consecutive integers?

(b) Count the n-digit numbers that contains only digits 1 and 2 and no two adjacent 1s?

(c) Count the number of ways of walk up n stairs where one step can be to the next step or you can skip one step.

(Hint: Derive a recurrence in each case.)

2:   Show that consecutive Fibonacci have greatest common divisor one.   That is $\gcd(F_n, F_{n-1}) = 1$ for all $n \geq 1$. In other words, they are relatively prime.

3:   Prove that Fibonacci numbers satisfy the following identities:

(a) $3F_n = F_{n+2} + F_{n-2}$ for $n \geq 2$.

(b) $4F_n = F_{n+2} + F_n + F_{n-2}$ for $n \geq 2$.

4:   Find generating functions for the following sequences (and simplify them - i.e. without infinite sums):
(a) $0, 0, 0, 4, -4, 4, -4, 4, -4, 4, -4, \ldots$
(b) $1, 4, 9, 16, 25, 36, 49, \ldots$
Use derivative, integral and/or decomposing one sequence into several simpler ones.

5:   Use generating functions to find the number of solutions to $z_1 + z_2 + z_3 + z_4 = 10$ where the z_i are nonnegative integers such that $z_1 \leq 4$, z_2 is odd, z_3 is prime (smallest prime is 2), and $z_4 \in \{1, 2, 3, 6, 8\}$. Use `http://wolframalpha.com` to expand the generating function.

6:   Find coefficient of x^k in the following generating functions:

(a) $\frac{1}{(1-x)(1-2x)}$

(b) $\frac{1}{(1-x)(1-x^2)}$