

Chapter 7.2 Generating Functions - Round II

Useful identities:

$$\begin{aligned}\frac{1-x^k}{1-x} &= \sum_{n=0}^{k-1} x^n \\ \frac{1}{1-x} &= \sum_{n=0}^{\infty} x^n \\ \frac{1}{(1-x)^k} &= \sum_{n=0}^{\infty} \binom{-k}{n} (-x)^n = \sum_{n=0}^{\infty} \binom{k+n-1}{n} x^n\end{aligned}$$

1: Find a generating function that counts how many ways is it possible to score 6 points in basketball.

2: Find generating function, where h_n counts the number of ways to score n points in basketball.

3: Determine the generating function for the number h_n of integral solutions of

$$2e_1 + 11e_2 + e_3 + 7e_4 = n,$$

where $0 \leq e_1$, $2 \leq e_2$, $0 \leq e_3 \leq 10$ and $1 \leq e_4 \leq 5$.

Use it to compute h_{31} . *Do not try to evaluate all h_n , just get the generating function and get h_{30} . But get a closed form function- no infinite sums or infinite products.*

4: (*Midterm practice*) Use generating function to compute the number of integer solutions to:

$$e_1 + e_2 + e_3 = 22$$

subject to $3 \leq e_1 \leq 8$, $6 \leq e_2 \leq 10$ and $2 \leq e_3 \leq 7$.