MATH-566 HW 5

Due **Sep 30** before class. Just bring it before the class and it will be collected there.

1: (*Farkas Lemma*)
Show that

$$A\mathbf{x} = \mathbf{b} \text{ has a non-negative solution iff } \forall \mathbf{y} \in \mathbb{R}^m \text{ with } \mathbf{y}^T A \geq \mathbf{0}^T \text{ implies } \mathbf{y}^T\mathbf{b} \geq 0$$

implies

$$A\mathbf{x} \leq \mathbf{b} \text{ has a non-negative solution iff } \forall \mathbf{y} \in \mathbb{R}^m,\ \mathbf{y} \geq \mathbf{0} \text{ with } \mathbf{y}^T A \geq \mathbf{0}^T \text{ implies } \mathbf{y}^T\mathbf{b} \geq \mathbf{0}.$$

2: (*Using simplex method*)
Convert the following program to equational form (add x_3, x_4, x_5) and solve it using the simplex method.

$$(P)\begin{cases} \text{maximize} & x_1 + 2x_2 \\ \text{subject to} & x_1 \leq 4 \\ & x_1 + x_2 \leq 5 \\ & -x_1 + x_2 \leq 1 \\ & x_1, x_2 \geq 0 \end{cases}$$

Use Bland's rule for selecting pivots. That is, pivot on variable with lowest possible index.

Use the last tableau to argue that the solution is indeed optimal.

Plot the set of feasible solutions of (P) and mark the solutions obtained after each iteration of the simplex method.

3: (*Starting simplex method*)
Suppose

$$(P)\begin{cases} \text{maximize} & \mathbf{c}^T\mathbf{x} \\ \text{subject to} & A\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0}. \end{cases}$$

Simplex method need an initial basic feasible solution. How to use the simplex method itself to obtain basic feasible solution? *(Find a new program that gives a basic feasible solution.)*

4: (*Fitting line as linear program*)
Some university in Iowa was measuring the loudness of the fan's screaming during the first touchdown of the local team. The measurements contain loudness in dB and the number of people at the stadium in thousands.

# fans	53	55	59	61.5	61.5
dB	90	94	95	100	105

Find a line $y = ax + b$ *best* fitting the data. There are several different notions of best fitting. Commonly used is least squares that is minimizing $\sum_i (ax_i + b - y_i)^2$. But big outliers move the result a lot (and it is troublesome to do it using linear programming). Use the one that minimizes the sum of differences. That is

$$\sum_i |ax_i + b - y_i|.$$

Write a linear program that solves the problem and solve it for the "measured" data.

(Fun facts: The Seattle Seahawks, who boast that their fans caused a small earthquake after a 2011 touchdown, acclaimed their crowds record 136.6-decibel noise level this September after an effort orchestrated by the fan group Volume 12. The loudest crowd roar at a sports stadium is 142.2 db and was achieved by fans of the Kansas City Chiefs, at Arrowhead Stadium in Kansas City, Missouri, on 29 September 2014.)