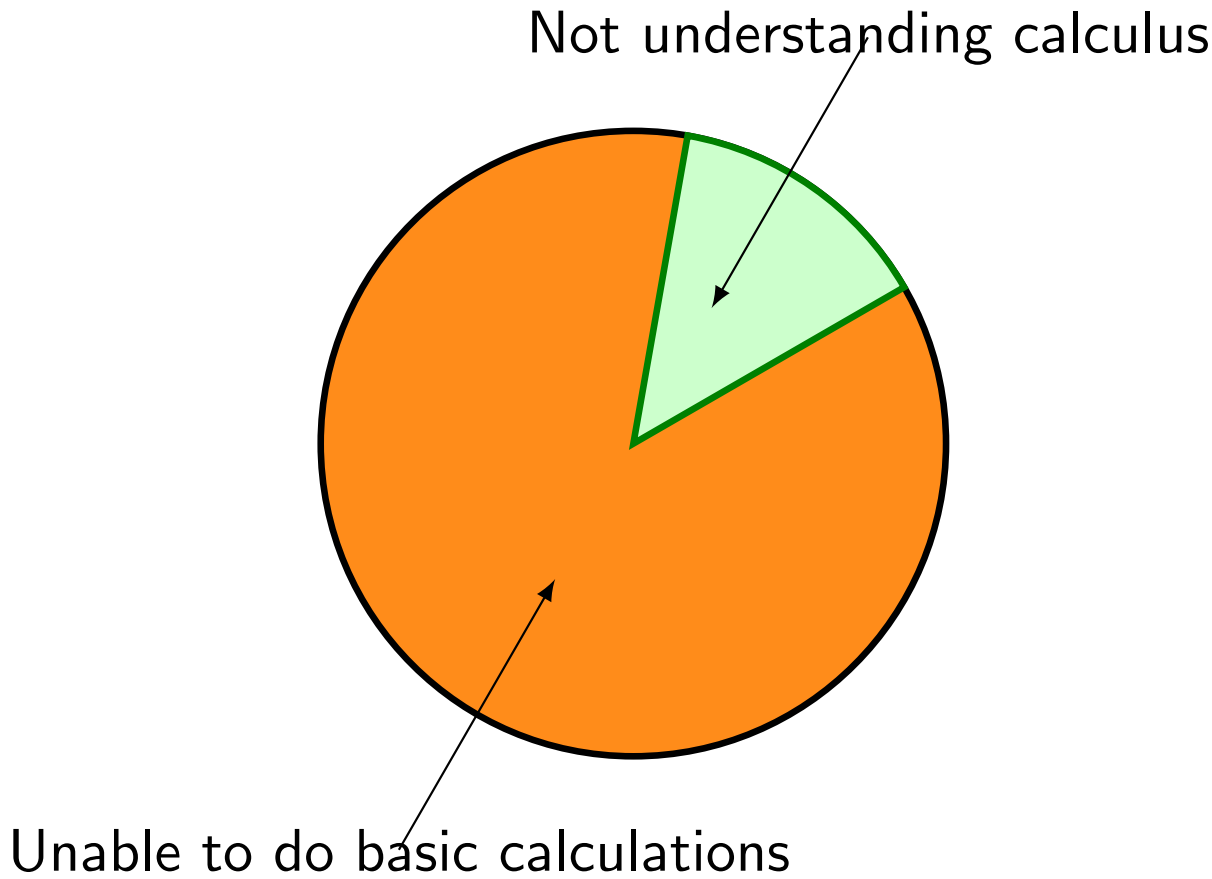
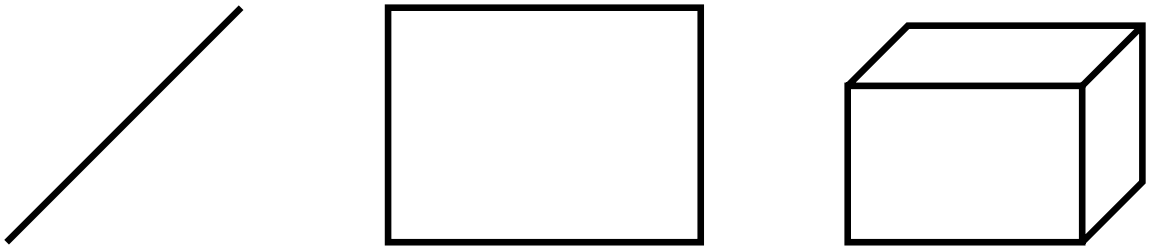
Why People Fail Calculus?



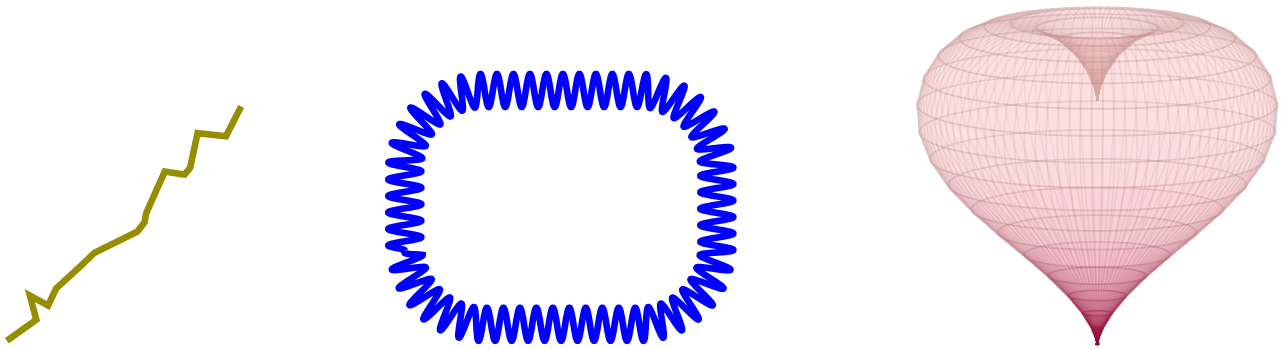
Take ALEKS placement test seriously.

Calculus on One Slide

We understand flat things very well



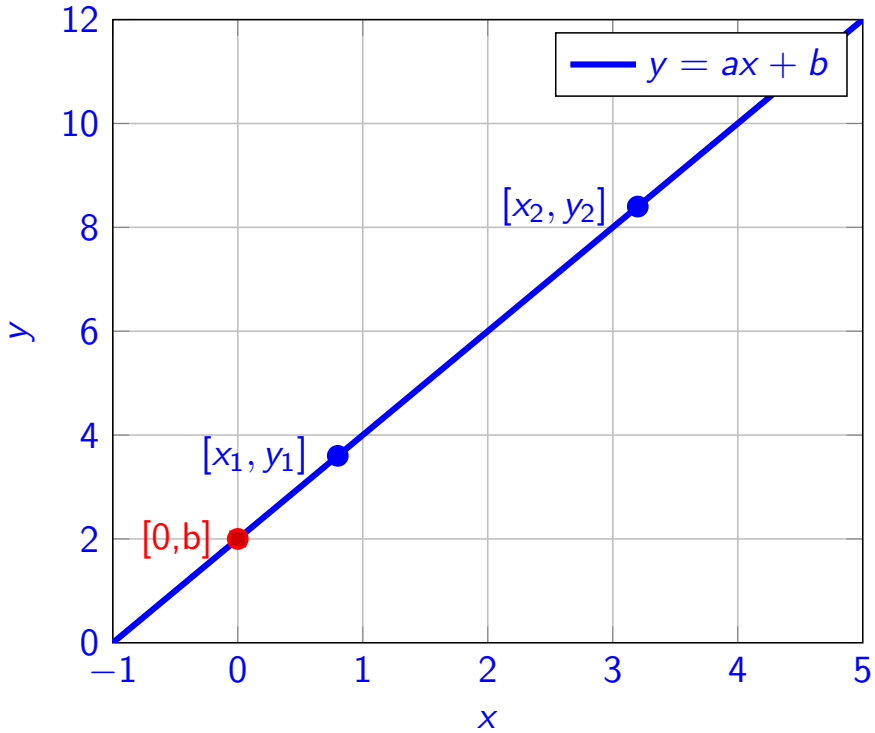
but world around us is NOT flat



we **approximate** things which are not flat by things that are flat.

Line in the plane $y = ax + b$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$



Slope measures *rate of change*.

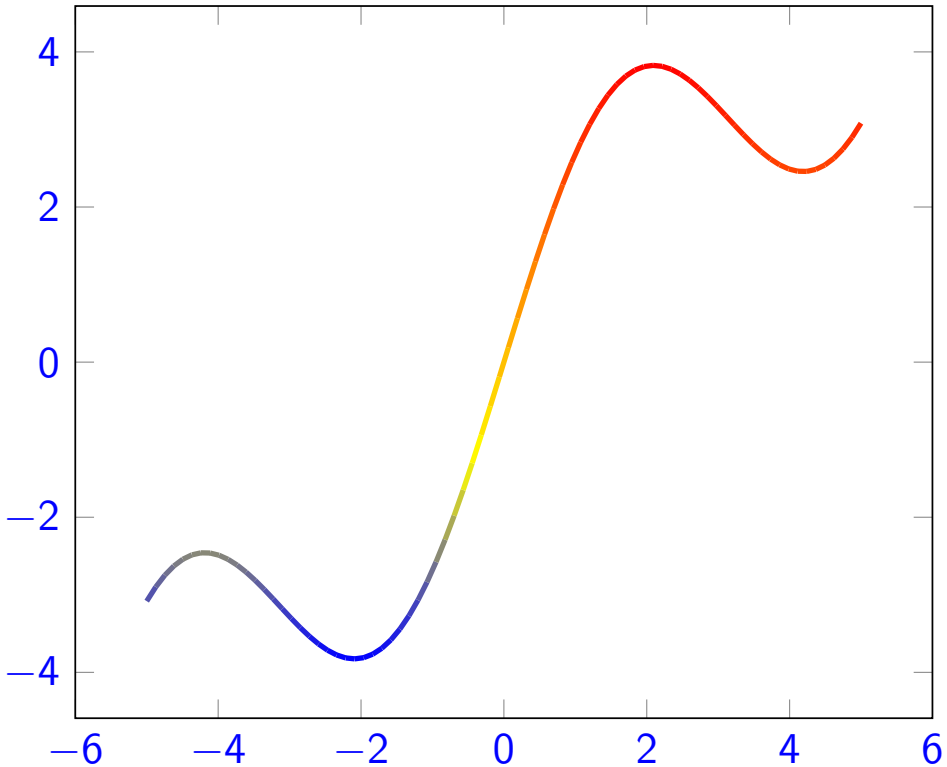
positive slope = positive rate of change
= going up/down

negative slope = negative rate of change
= going up/down

zero slope = no change
= going up/down

Example: Find the line passing trough points $[2, 1]$ and $[1, 3]$.

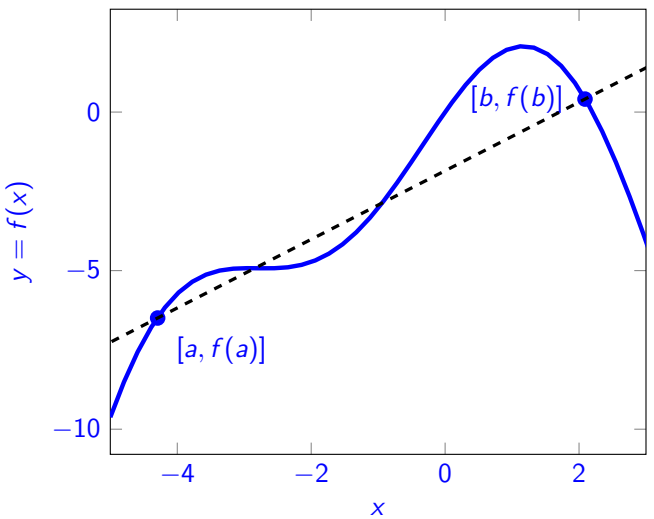
Differential Calculus - Tiny Changes - Earth is Flat



Average Rate of Change

The *average rate of change* for $y = f(x)$ from $x = a$ to $x = b$ is $\frac{f(b)-f(a)}{b-a} = \frac{\Delta y}{\Delta x}$.

Average rate of change is the slope of the *secant line* through $[a, f(a)]$ and $[b, f(b)]$.



Idea: Approximate f from a to b by a line.

Example: Find average rate of change for $y = 4x - 19$ from $x = \cos(3)$ to $x = \ln(\pi)$.

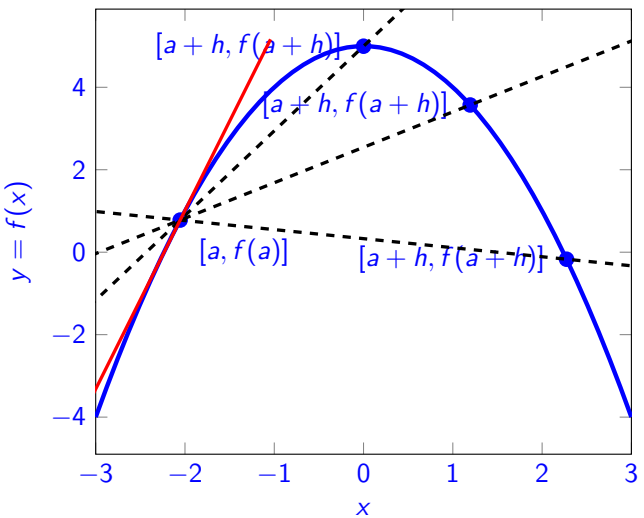
Example: Find average rate of change for $y = x^2 - 2$ from $x = 1$ to $x = 5$.

Instantaneous Rate of Change

The *instantaneous rate of change* for $y = f(x)$ at $x = a$ is the slope of *tangent* to $f(x)$ at $[a, f(a)]$.

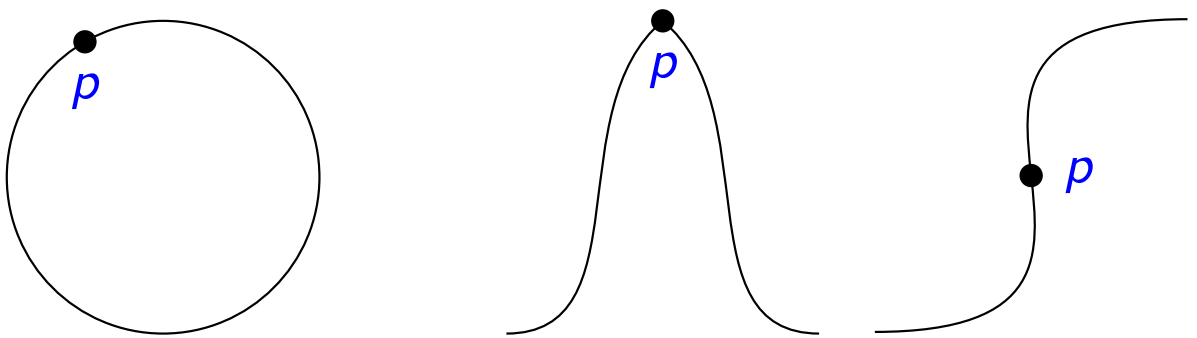
Approximate f from a to $a + h$ by a line and try to make h small (zero).

Example: Find the tangent line for $y = f(x) = 5 - x^2$ at $x = -2$.



Idea: Approximate $f(x)$ at a by a line.
Secant line goes to tangent line

Tangent Lines Do Not Always Exist



To compute the slope of tangent to $f(x)$ at $p = [a, f(a)]$ we study $f(a + h)$.
The slope is $\frac{\Delta y}{h} = \frac{f(a+h)-f(a)}{h}$ for $h = 0$ or $h \to 0$.

Chapter 2.1 Recap

- Line in the plane is $y = ax + b$
- Average rate of change of $f(x)$ from a to b is the slope of the secant
- Instantaneous rate of change of $f(x)$ is slope of tangent line
- Tangent line can be approximated by secant line
- Computing tangent line using $(x \to a + h)$ and $h \to 0$.
- Tangent line may not be defined