Genuine Quotes From Last Year

- I thought, because I took Calculus in high school, that I could do all the things an 'F student' does and still get an A.
- Three hours a week was not enough time for this instructor to teach and explain all the topics and I've taken calc I before.

Understanding is MORE Important Than Answer

- Understanding WHAT are you doing is important
- Answer alone is useless
- No need for calculators (solution steps matter)

What a Slope Should Be?

Recall: Slope of secant line is $\frac{f(b)-f(a)}{b-a}$.

Slope of tangent line would be slope of secant line with $a = b$.

$\frac{f(b)-f(a)}{b-a} = \frac{f(a)-f(a)}{a-a} = \frac{0}{0}$

Division by zero is bad!

$\frac{a}{b} = c$ whenever $a = bc$. Then $\frac{a}{0} = c$ gives $a = b0$.

Solution: Study $\frac{f(a+h)-f(a)}{h}$ where h is going to zero and find what it *should* be.

Example: Find tangent of $f(x) = \frac{1}{2}x^2 + 1$ at $a = 2$.

h	1	0.5	0.1	0.01	0.001
$\frac{f(2+h)-f(2)}{h}$	2.5	2.25	2.05	2.005	2.0005

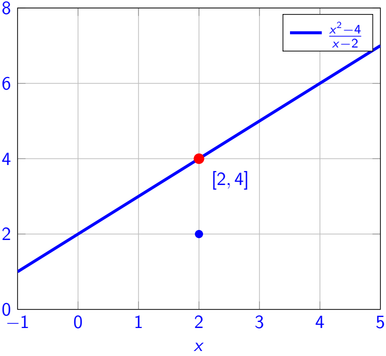
What *should* be $\frac{f(2+h)-f(2)}{h}$ for $h = 0$?

Limit of $f(x)$ at x_0

If $f(x)$ approaches L as x approaches x_0 we write

$$\boxed{\lim_{x\to x_0} f(x) = L}$$

approximate $f(x_0)$ by $f(x)$ around x_0
$f(x_0)$ may be undefined
maybe $f(x_0) \neq \lim_{x\to x_0} f(x)$
x is arbitrarily close to x_0 *but not* x_0

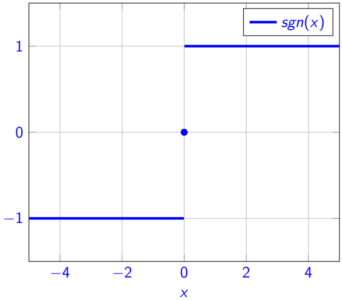


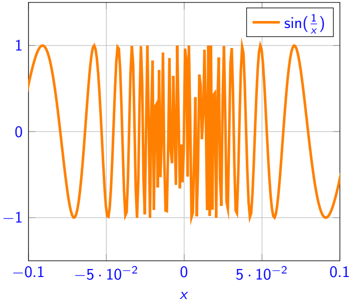
Example: Let $f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x \neq 2 \\ 2 & \text{if } x = 2 \end{cases}$.

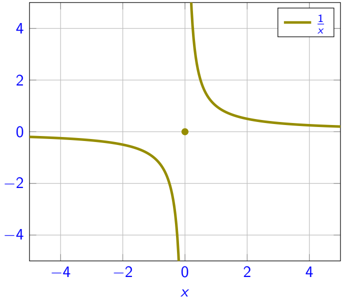
$\lim_{x\to 3} f(x) =$

$\lim_{x\to 2} f(x) =$

Limit May Not Exist







Basic Properties Of Limits

Let f and g be functions and $c \in \mathbb{R}$ a constant.

- $f(x) = c$. c is always near to c

$$\lim_{x \to a} c = c \qquad \lim_{x \to 3} 4 =$$

- $f(x) = x$. If x is close to a then $f(x)$ is close to a.

$$\lim_{x \to a} x = a \qquad \lim_{x \to \pi} x =$$

- Multiplying by scalar

$$\lim_{x \to a} c \cdot f(x) = c \cdot \lim_{x \to a} f(x) \qquad \lim_{x \to 7} 2x =$$

- Addition

$$\lim_{x \to a} (f(x) + g(x)) = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

$$\lim_{x \to 3} (2x + 4) =$$

If right hand side makes sense!

Limits are linear.

More Arithmetics With Limits

- Multiplication

$$\lim_{x\to a}(f(x)\cdot g(x)) = \lim_{x\to a} f(x)\cdot \lim_{x\to a} g(x)$$

$$\lim_{x\to 2}(x\cdot 2x) =$$

- Division

$$\lim_{x\to a}(f(x)/g(x)) = \lim_{x\to a} f(x)/\lim_{x\to a} g(x)$$

$$\lim_{x\to 2}\frac{2x+3}{x-9} =$$

- Power

$$\lim_{x\to a} f(x)^r = \left(\lim_{x\to a} f(x)\right)^r \qquad \lim_{x\to 4}(x+1)^3 =$$

If right hand side makes sense!

Everybody Loves Polynomials

Example:
$\lim_{x\to 1} x^3 - 3x + 1 =$

Polynomial: $f(x) = c_n x^n + c_{n-1}x^{n-1} + \cdots + c_1 x + c_0 = \sum_{j=0}^{n} c_j x^j$

$$\lim_{x\to a} f(x) = \lim_{x\to a} \left(c_n x^n + \cdots + c_1 x + c_0\right)$$

Example: $\lim_{x\to 3} \frac{x^2-2x+1}{x-1} =$

Example: $\lim_{x\to 1} \frac{x^2-2x+1}{x-1} =$

Polynomials are great! $\lim_{x\to a} f(x) = f(a)$. Easy to find limits!

Tricks For Evaluating $\frac{0}{0}$

Example: $\lim_{x\to 3} \frac{4(x-3)}{(x-3)} =$

Expanding/Factoring polynomials

Example:

$$\lim_{x\to \frac{1}{2}} \frac{2x^2+5x-3}{10x-5}$$

Tricks For Evaluating $\frac{0}{0}$

Recall: $(a+b)(a-b) =$

$$\boxed{\text{Multiplying by } 1 \ (= \tfrac{c}{c})}$$

Example:

$$\lim_{x \to 3} \frac{\sqrt{x+1} - 2}{\sqrt{x+6} - 3}$$

Tricks For Evaluating $\frac{0}{0}$

Recall: $\sin(x)^2 + \cos(x)^2 =$

Use Identities (trigonometry)

Example:

$$\lim_{x \to 0} \frac{\sin^2 x}{1 - \cos x}$$

Tricks For Evaluating $\frac{0}{0}$

Clever substitution

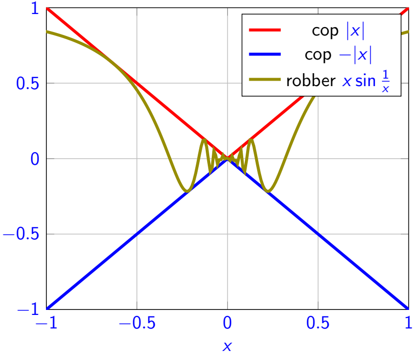
Example: $\lim_{z \to 0} \dfrac{\sqrt[3]{1+z} - 1}{z} =$

Squeeze Theorem - About Two Cops

Theorem (Sandwich, Squeeze, *About 2 cops*)

If $g(x) \leq f(x) \leq h(x)$ *near c and* $\lim_{x\to c} g(x) = \lim_{x\to c} h(x) = L$, *then* $\lim_{x\to c} f(x) = L$.

Example: Compute $\lim_{x\to 0} x \sin\left(\frac{1}{x}\right)$



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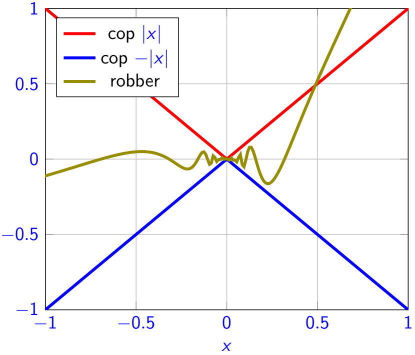

Example: Compute

$$\lim_{x\to 0} (e^x - 1)x \sin\left(\frac{1}{x}\right)$$

Chapter 2.2 Recap

- Limit of $f(x)$ at a, what $f(x)$ should be?
- Limit may be undefined.
- Limits are linear.
- It is also easy to multiply, divide, take power.
- Limits of polynomials are easy.
- Tips and Tricks for $\frac{0}{0}$.
- 2 cops Theorem