Why Are We Learning Calculus I?

- It is prerequisite?
- It is mandatory?

What Does Approaches Means?

If $f(x)$ *approaches* L as x *approaches* a we write

$$\boxed{\lim_{x \to a} f(x) = L}$$

What is the *distance* between x and y?

As x gets close to a then $f(x)$ gets close to L.

As decreases then decreases.

We want $f(x)$ close to L for all x very close to a.

As is small for all x where is tiny.

Think of ε and δ as (very) small distances.

We want $f(x)$ being ε-close to L for all x being δ-close to a.

for all x where

What Does Approaches Means with ε and δ?

If $f(x)$ *approaches* L as x *approaches* a we write

$$\boxed{\lim_{x\to a} f(x) = L}$$

We want $f(x)$ being ε-close to L for all x being δ-close to a.

$|f(x) - L| < \varepsilon$ for all $0 < |x - a| < \delta$

We can make $|f(x) - L| < \varepsilon$ for any $\varepsilon > 0$ as long as $0 < |x - a| < \delta$ for some $\delta > 0$

$\lim_{x\to a} f(x) = L$ if

$$\boxed{\text{for every } \varepsilon > 0 \text{ exists } \delta > 0 \text{ so if } 0 < |x - a| < \delta \text{ then } |f(x) - L| < \varepsilon}$$

Game approach:

Game approach?

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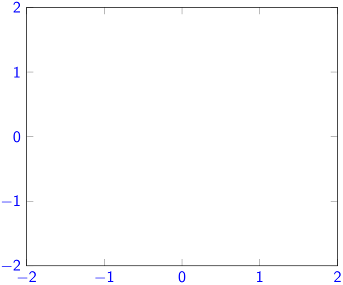
$$\boxed{\text{for every } \varepsilon > 0 \text{ exists } \delta > 0 \text{ so if } 0 < |x - a| < \delta \text{ then } |f(x) - L| < \varepsilon}$$

Game approach: You give me $\varepsilon > 0$ and I will find you a good $\delta > 0$



Example: Is it true that $\lim_{x \to 0} f(x) = 0$

for $f(x) = \begin{cases} x - 1 & x < 0 \\ 0 & x = 0 \\ x + 1 & x > 0 \end{cases}$

ε, δ Examples

Example: Show $\lim_{x \to a} c = c$.

Example: Show $\lim_{x \to a} 2 + x^2 = \quad$.

Example: Show $\lim_{x \to a} x = x$.

More Rigorous Sum Of Limits

Example:
Show that if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$ then $\lim_{x\to a} f(x) + g(x) = L + M$.

Recall: For given $\varepsilon > 0$ find $\delta > 0$ such that $|x - a| < \delta$ implies $|f(x) + g(x) - L - K| < \varepsilon$.

Trick: $|f(x) + g(x) - L - K| \leq |f(x) - L| + |g(x) - M|$
(triangle inequality)

$\lim_{x\to a} f(x) = L$ implies

$\lim_{x\to a} g(x) = M$ implies

Pick $\delta =$. Then for all $|x - a| < \delta$ holds

Chapter 2.3 Recap

- There exists a rigorous definition of a limit.
- Can be played as a game: for given ε find δ.
- Not part of midterm or final.
- More about this in MATH-201 Introduction to proofs and MATH-414 (Analysis I)