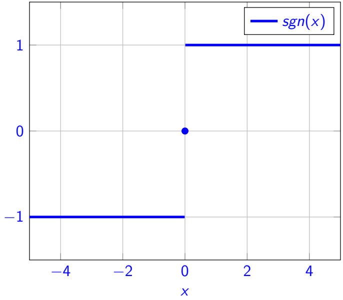
Chapter 2.4 - One-sided limits

Motivation for one-sided limit



Recall: $\lim\limits_{x \to 0} sgn(x)$ is not defined.

If we look at $\lim\limits_{x \to 0} sgn(x)$ *only* for $x > 0$ then the limit *could* be 1.

If we look at $\lim\limits_{x \to 0} sgn(x)$ *only* for $x < 0$ then the limit *could* be -1.

One-sided limit definition

If $f(x)$ approaches L as x approaches a from *the right (i.e. $x > a$)* we write

$$\lim_{x \to a^+} f(x) = L$$

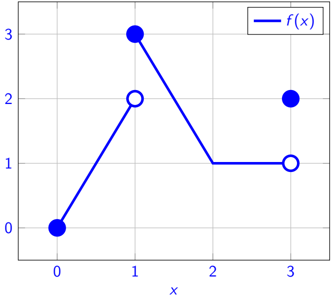
If $f(x)$ approaches L as x approaches a from *the left (i.e. $x < a$)* we write

$$\lim_{x \to a^-} f(x) = L$$

$\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^+} f(x) = L$ and $\lim_{x \to a^-} f(x) = L$

Simple example for limits

Example: Compute limits and values for the $f(x)$ defined on $[0, 3]$.



- $\lim_{x \to 1^-} f(x) =$
- $\lim_{x \to 1^+} f(x) =$
- $f(1) =$
- $\lim_{x \to 2} f(x) =$
- $f(3) =$
- $\lim_{x \to 3^-} f(x) =$
- $\lim_{x \to 3^+} f(x) =$

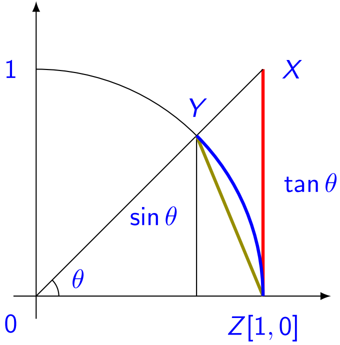
$\sin(x)$ behaves like the line $y = x$ around 0.

$$\lim_{x \to 0^+} \frac{\sin x}{x} = 1$$

Assume $\pi/2 > \theta > 0$. Then

$$\text{area } \Delta 0ZY < \text{area sector } 0ZY < \text{area } \Delta 0ZX$$

area $\Delta 0ZY =$
area sector $0ZY =$
area $\Delta 0ZX =$



Examples using $\lim_{x\to 0} \frac{\sin x}{x} = 1$

- $\lim_{h\to 0} \frac{h}{\sin(3h)} =$

- $\lim_{x\to 0} \frac{\tan(2x)}{x} =$

- $\lim_{\theta\to 0} \frac{\cos(\theta) - 1}{\theta} =$

- $\lim_{\theta\to 0} \frac{\cos(\theta) - 1}{\sin(2\theta)} =$

Chapter 2.4 Recap

- One-sided limit is approaching x only from one side
- One-sided limits are same IFF limit exists
- $\lim_{x \to 0} \frac{\sin x}{x} = 1$