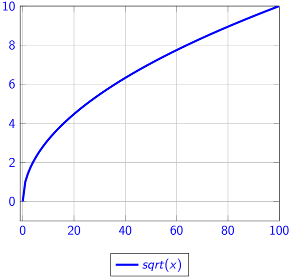
Chapter 3.11: Linearization and Differentials

Motivational Question

Example: Estimate $\sqrt{65}$ without using calculator.

Approximate $\sqrt{x}$ using linear function!



Linearization

If $y = f(x)$ is differentiable at $x = a$, then

$$\boxed{L(x) = f(a) + f'(a)(x - a)}$$

is the line tangent to f at the point $(a, f(a))$. We call this the *linearization* of $f(x)$ at $x = a$.

Example: Find the linearization of the following functions at the specified point:

- $f(x) = x^3 - 2x + 3$ at $a = 2$
- $f(x) = x + 1/x$ at $a = 1$
- $f(x) = \tan(x)$ at $a = \pi$

Approximation Examples $\boxed{L(x) = f(a) + f'(a)(x - a)}$

Find approximations for the following values:

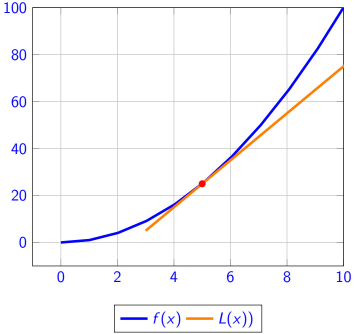
1. $\sqrt{1.1}$ exact value is ≈ 1.0488088481.

2. $\sin(.01)$ exact value is ≈ 0.00999983.

Approximation and Small Changes

Let $L(x)$ be a linearization of $f(x)$ at point a.

If $x = a$ is changed by Δx, what is change ΔL in $L(x)$?



Recall
$L(x) = f(a) + f'(a)(x - a)$.

Differentials

Leibniz says that we can treat $\frac{dy}{dx}$ like a fraction and write a formula for dy in terms of dx and $f'(x)$. This is known as the *differential* form of the derivative.

Find the differential form of the derivative of the following
(use dy and dx for derivative of x and y):

1. $y = \sqrt{1 - x^2}$

2. $xy^2 - 4x^{3/2} - y = 0$

3. Use implicit differentiation on $xy^2 - 4x^{3/2} - y = 0$

Approximation With Differentials Example

1. Approximate $\sqrt[3]{1.009}$ using differentials and $f(a+dx) \approx f(a) + dy$

2. Approximate $\sqrt[3]{1.009}$ using $L(x) = f(a) + f'(a)(x-a)$.

$\sqrt[3]{1.009} = 1.0029910447317696162055702742437385366236726998841849434\ldots$