Chapter 3.3: Differentiation Rules

Basic Functions (compute them once and for all)

$\frac{d}{dx}\left[c\right] =$

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$\frac{d}{dx}\left[x\right] =$

$$\frac{d}{dx}\left[x\right] =$$

$\frac{d}{dx}\left[x^n\right] =$

$$\frac{d}{dx}\left[x^n\right] =$$

$\frac{d}{dx}\left[e^x\right] =$

Fact: $\lim_{h\to 0} \frac{e^h - 1}{h} = 1$; $e = 2.718281828459\ldots$

$$\frac{d}{dx}\left[e^x\right] =$$

Combining Functions I

Pulling out constants

$$\boxed{\frac{d}{dx}\Big[c \cdot f(x)\Big] = c \cdot \frac{d}{dx}\Big[f(x)\Big]}$$

$$\frac{d}{dx}\Big[cf(x)\Big] =$$

$$= \lim_{h\to 0} \frac{cf(x+h) - cf(x)}{h}$$

$$=$$

Example: $\frac{d}{dx}\Big[3x^2\Big] =$

Separating over sums

$$\boxed{\frac{d}{dx}\Big[f(x) + g(x)\Big] = \frac{d}{dx}\Big[f(x)\Big] + \frac{d}{dx}\Big[g(x)\Big]}$$

$$\frac{d}{dx}\Big[f(x) + g(x)\Big] =$$

$$= \lim_{h\to 0} \frac{[f(x+h) + g(x+h)] - [f(x) + g(x)]}{h}$$

$$=$$

Example: $\frac{d}{dx}\Big[x^3 + x\Big] =$

Examples

Example: Find $\dfrac{d}{dx}\left[\dfrac{2}{x}+e^{100}\right]$

Example: Find tangent line at $x=1$ to function $f(x)=x^3+\sqrt{x}-\frac{2}{e}e^x$.

Example: Find all x so that the tangent lines to $y=x^3-12x+17$ are horizontal.

Product Rule

$$\boxed{\frac{d}{dx}\Big[f(x)g(x)\Big] = \frac{d}{dx}\Big[f(x)\Big]g(x) + f(x)\frac{d}{dx}\Big[g(x)\Big]}$$

$$\begin{aligned}
\frac{d}{dx}\Big[f(x)g(x)\Big] &= \lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x)}{h} \\
&= \lim_{h\to 0}\frac{f(x+h)g(x+h)\overbrace{-f(x)g(x+h)+f(x)g(x+h)}^{=}-f(x)g(x)}{h} \\
&= \lim_{h\to 0}\left(\underbrace{\frac{f(x+h)-f(x)}{h}}_{=}g(x+h)+f(x)\underbrace{\frac{g(x+h)-g(x)}{h}}_{=}\right) \\
&= f'(x)g(x)+f(x)g'(x)
\end{aligned}$$

Example: Find $\frac{d}{dx}\left(x^{2/3}e^{x}\right) =$

Reciprocal and Quotient Rules

Reciprocal rule

$$\boxed{\frac{d}{dx}\left[\frac{1}{f(x)}\right] = \frac{-f'(x)}{f(x)^2}}$$

Will be obvious after Chain rule in Section 3.6.

Quotient rule

$$\boxed{\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}}$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{d}{dx}\left[f(x)\cdot\frac{1}{g(x)}\right] =$$

Examples for Quotient Rule

$$\boxed{\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}}$$

Example: Find $\dfrac{d}{dx}\left[\dfrac{x^2}{x+2}\right]$

Example: Find $\dfrac{d}{dx}\left[\dfrac{\sqrt{x}+x^2}{3x^3+x\cdot e^x}\right]$

Higher Order Derivatives

Derivatives are functions so we can take derivatives of derivatives, and so on.

$$y = f(x) = \text{ function}$$

$$y' = \frac{dy}{dx} = f'(x) = \text{ first derivative}$$

$$y'' = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d^2y}{dx^2} = f''(x) = \text{ second derivative}$$

$$y^{(n)} = \frac{d^ny}{dx^n} = f^{(n)}(x) = n^{\text{th}} \text{ derivative}$$

Example: Compute derivatives of $y = 2x^3 - x^2 + 4x + 3$

$$\frac{dy}{dx} =$$

$$\frac{d^2y}{dx^2} =$$

$$\frac{d^3y}{dx^3} =$$

$$\frac{d^4y}{dx^4} =$$

$$\frac{d^5y}{dx^5} =$$

More Examples

Example: Find derivatives of
$f(x) = (7 - 2x) \cdot (5 + x^3)^{-1}$

$f(x) = e^{-x}$

$f(x) = e^{2x}$

$$f(x) = \frac{1 - 2x + 4\sqrt{x}}{x}$$

Example: Find second derivative of $f(x) = \frac{x^3+7}{x}$.

Chapter 3.3 Recap

$$\frac{d}{dx}\left[x^r\right] = rx^{r-1} \qquad\qquad \frac{d}{dx}\left[e^x\right] = e^x$$

$$\boxed{\frac{d}{dx}\left[c \cdot f(x)\right] = c \cdot \frac{d}{dx}\left[f(x)\right]}$$

$$\boxed{\frac{d}{dx}\left[f(x) + g(x)\right] = \frac{d}{dx}\left[f(x)\right] + \frac{d}{dx}\left[g(x)\right]}$$

$$\boxed{\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}}$$

$\frac{d^2y}{dx^2}\,[f(x)]$ is the second derivative