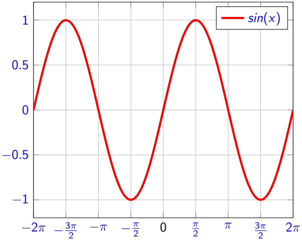
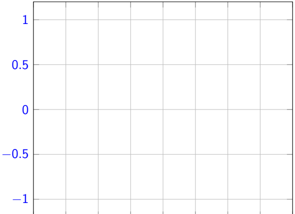
Chapter 3.5: Derivatives of Trigonometric Functions

Derivative of $\sin(x)$

$$\sin(x+h) = \sin(x)\cos(h) + \cos(x)\sin(h)$$

$$\lim_{h\to 0} \frac{\sin(h)}{h} = \qquad\qquad \lim_{h\to 0} \frac{\cos(h)-1}{h} =$$



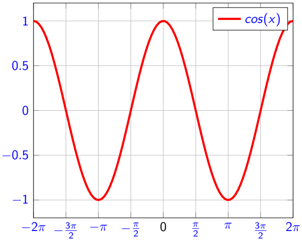


$$\frac{d}{dx}\Big[\sin(x)\Big]$$

$$= \lim_{h\to 0} \frac{\sin(x+h)-\sin(x)}{h}$$

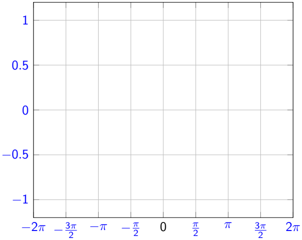
Derivative of $\cos(x)$

$$\cos(x+h) = \cos(x)\cos(h) - \sin(x)\sin(h)$$



$$\frac{d}{dx}\Big[\cos(x)\Big]$$

$$= \lim_{h\to 0} \frac{\cos(x+h) - \cos(x)}{h}$$



Examples for sin and cos

$$\boxed{\frac{d}{dx}[\sin(x)] = cos(x)} \qquad \boxed{\frac{d}{dx}[\cos(x)] = -sin(x)}$$

1. $y = 3/x + 5\sin(x)$

2. $f(x) = e^x \sin(x)$

3. $f(x) = \sin(x)\cos(x)$

4. $\dfrac{d}{dx}\left[\dfrac{\cos(x)}{1 - sin(x)}\right]$

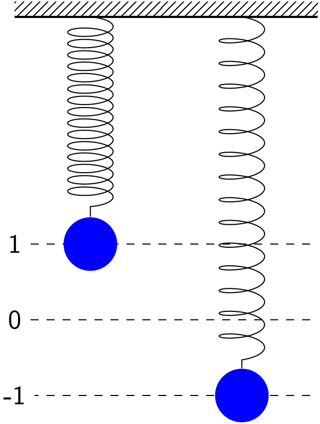
Higher Derivatives

$$\boxed{\frac{d}{dx}[\sin(x)] = cos(x)} \qquad \boxed{\frac{d}{dx}[\cos(x)] = -sin(x)}$$

Find $\dfrac{d^{999}}{dx^{999}}\left[\sin(x)\right]$

Simple Harmonic Motion

A weight on a spring is released from height one. The position of weight in time t is given by $s(t) = cos(t)$.



$v(t) =$

$a(t) =$

Example: In a damped motion the position is given by $s(t) = e^{-t}\cos(t)$. Compute the limit of acceleration as $t \to \infty$.

Other Trigonometric Functions

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \qquad \cot(x) = \frac{\cos(x)}{\sin(x)} \qquad \sec(x) = \frac{1}{\cos(x)} \qquad \csc(x) = \frac{1}{\sin(x)}$$

$$\frac{d}{dx}\Big[\tan(x)\Big] =$$

$$\frac{d}{dx}\Big[\cot(x)\Big] =$$

$$\frac{d}{dx}\Big[\sec(x)\Big] =$$

$$\frac{d}{dx}\Big[\csc(x)\Big] =$$

Chapter 3.5 Recap

- $$\boxed{\frac{d}{dx}[\sin(x)] = cos(x)}$$

- $$\boxed{\frac{d}{dx}[\cos(x)] = -sin(x)}$$

- Derivatives of other trigonometric functions follow from sin and cos.