Chapter 3.6: Chain Rule

Rules For Derivates

$$\frac{d}{dx}(1) = 0$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(c \cdot f(x)) = c \cdot f'(x)$$

$$\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$$

$$\frac{d}{dx}(f(x) \cdot g(x)) = f(x)' \cdot g'(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

Chain rule:

$$\frac{d}{dx}(f(g(x))) =$$

First Example and Idea

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x)) \cdot g'(x)$$

- Differentiate the outside
- Plug the inside
- Multiply by derivative on inside

Proof idea:

$$\frac{d}{dx}\left(f(g(x))\right) =$$

Example:

$$\frac{d}{dx}\left[\sin(x^3)\right] =$$

$$\frac{d}{dx}\left[\cos(x^2) - e^{x^2}\right] =$$

Examples

$$\frac{d}{dx}\left(e^{31x}\right) =$$

$$\frac{d}{dx}\left(\left(e^{x}\right)^{31}\right) =$$

$$\frac{d}{dx}\left(\sin(\cos(\sqrt{x}))\right) =$$

$$\frac{d}{dx}\left(|x|\right) =$$

$$\frac{d}{dx}\left(e^{x\sqrt{x^6+1}}\right) =$$

Reciprocal and Quotient Rules Using Chain Rule

Reciprocal rule

$$\frac{d}{dx}\left[\frac{1}{f(x)}\right] = \frac{-f'(x)}{f(x)^2}$$

$$\frac{d}{dx}\left[\frac{1}{f(x)}\right] =$$

Quotient rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] =$$

Magic Table

Given the following table

x=	1	2	3	4
f(x)=	3	4	2	1
f'(x)=	4	3	1	2
g(x)=	2	4	3	1
g'(x)=	2	1	4	3

Determine $h'(x)$ and $k'(x)$
for $x = 1, 2, 3, 4$ given that
$h(x) = f(g(x))$ and $k(x) = g(f(x))$.

Chapter 3.6 Recap

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x)) \cdot g'(x)$$